

Friderikos, Orestis; Olive, Marc; Baranger, Emmanuel; Sagris, Dimitris; David, Constantine
A non-intrusive space-time interpolation from compact Stiefel manifolds of parametrized rigid-viscoplastic FEM problems. (English) Zbl 1478.74080
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Summary: This work aims to interpolate parametrized reduced order model (ROM) basis constructed via the proper orthogonal decomposition (POD) to derive a robust ROM of the system's dynamics for an unseen target parameter value. A novel non-intrusive space-time (ST) POD basis interpolation scheme is proposed, for which we define ROM spatial and temporal basis *curves on compact Stiefel manifolds*. An interpolation is finally defined on a *mixed part* encoded in a square matrix directly deduced using the spacial part, the singular values and the temporal part, to obtain an interpolated snapshot matrix, keeping track of accurate space and temporal eigenvectors. Moreover, in order to establish a well-defined curve on the compact Stiefel manifold, we introduce a new procedure, the so-called oriented SVD. Such an oriented SVD produces unique right and left eigenvectors for generic matrices, for which all singular values are distinct. It is important to notice that the ST POD basis interpolation does not require the construction and the subsequent solution of a reduced-order FEM model as classically is done. Hence it is avoiding the bottleneck of standard POD interpolation which is associated with the evaluation of the nonlinear terms of the Galerkin projection on the governing equations. As a proof of concept, the proposed method is demonstrated with the adaptation of rigid-thermoviscoplastic finite element ROMs applied to a typical nonlinear open forging metal forming process. Strong correlations of the ST POD models with respect to their associated high-fidelity FEM counterpart simulations are reported, highlighting its potential use for near real-time parametric simulations using off-line computed ROM POD databases.

MSC:

- 74S05 Finite element methods applied to problems in solid mechanics
- 74C10 Small-strain, rate-dependent theories of plasticity (including theories of viscoplasticity)
- 53Z05 Applications of differential geometry to physics

Keywords:

proper orthogonal decomposition; parametrized model order reduction; Grassmann manifold; Stiefel manifold; oriented singular value decomposition; rigid-viscoplastic finite element method

Software:

Mulprec

Full Text: [DOI](#) [arXiv](#)

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