

Hillman, Michael; Lin, Kuan-Chung

Nodally integrated thermomechanical RKPM. II: Generalized thermoelasticity and hyperbolic finite-strain thermoplasticity. (English) [Zbl 1478.74085](#)

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Summary: In this two-part paper, a stable and efficient nodally-integrated reproducing kernel particle method (RKPM) approach for solving the governing equations of generalized thermomechanical theories is developed. Part I [the authors, *ibid.* 68, No. 4, 795–820 (2021, [Zbl 1478.74084](#))] investigated quadrature in the weak form using classical thermoelasticity as a model problem, and a stabilized and corrected nodal integration was proposed. In this sequel, these methods are developed for generalized thermoelasticity and generalized finite-strain plasticity theories of the hyperbolic type, which are more amenable to explicit time integration than the classical theories. Generalized thermomechanical models yield finite propagation of temperature, with a so-called second sound speed. Since this speed is not well characterized for common engineering materials and environments, equating the elastic wave speed with the second sound speed is investigated to obtain results close to classical thermoelasticity, which also yields a uniform critical time step. Implementation of the proposed nodally integrated RKPM for explicit analysis of finite-strain thermoplasticity is also described in detail. Several benchmark problems are solved to demonstrate the effectiveness of the proposed approach for thermomechanical analysis.

MSC:

[74S99](#) Numerical and other methods in solid mechanics

[74F05](#) Thermal effects in solid mechanics

[74B05](#) Classical linear elasticity

[74C15](#) Large-strain, rate-independent theories of plasticity (including nonlinear plasticity)

[74J99](#) Waves in solid mechanics

Keywords:

meshfree reproduced kernel particle method; generalized thermoelasticity; finite-strain thermoplasticity; nodal integration; elastic wave; second sound speed

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