

Dai, Xiaoqiang; Han, Jiangbo; Lin, Qiang; Tian, Xueting
Anomalous pseudo-parabolic Kirchhoff-type dynamical model. (English) Zbl 1479.35544
Adv. Nonlinear Anal. 11, 503-534 (2022).

Summary: In this paper, we study an anomalous pseudo-parabolic Kirchhoff-type dynamical model aiming to reveal the control problem of the initial data on the dynamical behavior of the solution in dynamic control system. Firstly, the local existence of solution is obtained by employing the Contraction Mapping Principle. Then, we get the global existence of solution, long time behavior of global solution and blowup solution for $J(u_0) \leq d$, respectively. In particular, the lower and upper bound estimates of the blowup time are given for $J(u_0) < d$. Finally, we discuss the blowup of solution in finite time and also estimate an upper bound of the blowup time for high initial energy.

MSC:

- 35K70 Ultraparabolic equations, pseudoparabolic equations, etc.
- 35B40 Asymptotic behavior of solutions to PDEs
- 35B44 Blow-up in context of PDEs
- 35K20 Initial-boundary value problems for second-order parabolic equations
- 35K59 Quasilinear parabolic equations
- 35R11 Fractional partial differential equations

Keywords:

pseudo-parabolic Kirchhoff-type equation; global existence

Full Text: [DOI](#)

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