

**Javaherian, Ashkan; Cox, Ben****Ray-based inversion accounting for scattering for biomedical ultrasound tomography.** (English) [Zbl 1478.78032](#)

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**MSC:**

- 78A46 Inverse problems (including inverse scattering) in optics and electromagnetic theory
- 74A45 Theories of fracture and damage
- 74A05 Kinematics of deformation
- 76Q05 Hydro- and aero-acoustics
- 74J20 Wave scattering in solid mechanics
- 65K10 Numerical optimization and variational techniques
- 65M80 Fundamental solutions, Green's function methods, etc. for initial value and initial-boundary value problems involving PDEs
- 65D25 Numerical differentiation
- 35J05 Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation
- 92C55 Biomedical imaging and signal processing
- 94A08 Image processing (compression, reconstruction, etc.) in information and communication theory

**Keywords:**

ultrasound tomography; sound speed; ray-based inversion; scattering; Born approximation

**Software:**

k-Wave

**Full Text:** [DOI](#) [arXiv](#)**References:**

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