

Zamorano, S.

Approximate controllability from the exterior for a nonlocal Sobolev-Galpern type equation.

(English) [Zbl 1478.35229](#)

Math. Notes 110, No. 4, 609-622 (2021).

Summary: In this paper, we study the approximate control problem from the exterior of a nonlocal equation of Sobolev-Galpern type, specifically the Barenblatt-Zhel'tov-Kochina equation, involving the fractional Laplace operator of order $s \in (0, 1)$. We prove that the system under consideration is approximate controllable at any time $T > 0$.

MSC:

[35R11](#) Fractional partial differential equations

[35B60](#) Continuation and prolongation of solutions to PDEs

[35K20](#) Initial-boundary value problems for second-order parabolic equations

[35K70](#) Ultraparabolic equations, pseudoparabolic equations, etc.

[93B05](#) Controllability

Keywords:

fractional Laplace operator; Sobolev-Galpern type equation; exterior control problem; Barenblatt-Zhel'tov-Kochina equation; unique continuation property; approximate controllability

Full Text: [DOI](#)

References:

- [1] Warma, M., Approximate controllability from the exterior of space-time fractional diffusive equations, *SIAM J. Control Optim.*, 57, 3, 2037-2063 (2019) · [Zbl 1421.93024](#) · [doi:10.1137/18M117145X](#)
- [2] Barenblatt, G.; Zhel'tov, I.; Kochina, I., Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks, *J. Appl. Math. Mech.*, 24, 5, 1286-1303 (1960) · [Zbl 0104.21702](#) · [doi:10.1016/0021-8928\(60\)90107-6](#)
- [3] Ting, T. W., Certain nonsteady flows of second-order fluids, *Arch. Rational Mech. Anal.*, 14, 1-26 (1963) · [Zbl 0139.20105](#) · [doi:10.1007/BF00250690](#)
- [4] Benjamin, T.; Bona, J.; Mahony, J., Model equations for long waves in nonlinear dispersive systems, *Phil. Trans. R. Soc. Lond. A*, 272, 1220, 47-78 (1972) · [Zbl 0229.35013](#) · [doi:10.1098/rsta.1972.0032](#)
- [5] Chaves-Silva, F. W.; Souza, D. A., On the controllability of some equations of Sobolev-Galpern type, *J. Differential Equations*, 268, 4, 1633-1657 (2020) · [Zbl 1429.93037](#) · [doi:10.1016/j.jde.2019.09.005](#)
- [6] Tao, Q.; Gao, H.; Yao, Z., Boundary controllability of a pseudoparabolic equation, *Abstract and Applied Analysis*, 0, 1-6 (2013) · [Zbl 1291.93047](#) · [doi:10.1155/2013/483053](#)
- [7] Zuazua, E., *Controllability of Partial Differential Equations* (2006), Spain: Castro Urdiales, Spain
- [8] Louis-Rose, C.; Warma, M., Approximate controllability from the exterior of space-time fractional wave equations, *Appl. Math. Optim.*, 0, 1-44 (2018)
- [9] Warma, M.; Zamorano, S., Null controllability from the exterior of a one-dimensional nonlocal heat equation, *Control & Cybernetics*, 48, 3, 417-438 (2019) · [Zbl 1465.93020](#)
- [10] Nezza, E. Di; Palatucci, G.; Valdinoci, E., Hitchhiker's guide to the fractional Sobolev spaces, *Bull. Sci. Math.*, 136, 5, 521-573 (2021) · [Zbl 1252.46023](#) · [doi:10.1016/j.bulsci.2011.12.004](#)
- [11] Grisvard, P., *Elliptic Problems in Nonsmooth Domains* (2011), Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA · [Zbl 1231.35002](#) · [doi:10.1137/1.9781611972030](#)
- [12] Jonsson, A.; Wallin, H., Function spaces on subsets of \mathbb{R}^n , *Math. Rep.*, 2, 1, xiv+221 (1984) · [Zbl 0875.46003](#)
- [13] Warma, M., The fractional relative capacity and the fractional Laplacian with Neumann and Robin boundary conditions on open sets, *Potential Anal.*, 42, 2, 499-547 (2015) · [Zbl 1307.31022](#) · [doi:10.1007/s11118-014-9443-4](#)
- [14] Bogdan, K.; Burdzy, K.; Chen, Z.-Q., Censored stable processes, *Probab. Theory Related Fields*, 127, 1, 89-152 (2003) · [Zbl 1032.60047](#) · [doi:10.1007/s00440-003-0275-1](#)
- [15] Caffarelli, L. A.; Roquejoffre, J.-M.; Sire, Y., Variational problems for free boundaries for the fractional Laplacian, *J. Eur. Math. Soc.*, 12, 5, 1151-1179 (2010) · [Zbl 1221.35453](#) · [doi:10.4171/JEMS/226](#)
- [16] Gal, C. G.; Warma, M., Bounded solutions for nonlocal boundary value problems on Lipschitz manifolds with boundary, *Adv.*

Nonlinear Stud., 16, 3, 529-550 (2016) · Zbl 1342.35435 · doi:10.1515/ans-2015-5033

- [17] Gal, C. G.; Warma, M., Nonlocal transmission problems with fractional diffusion and boundary conditions on nonsmooth interfaces, *Comm. Partial Differential Equations*, 42, 4, 579-625 (2017) · Zbl 1373.35331 · doi:10.1080/03605302.2017.1295060
- [18] Gal, C. G.; Warma, M., On some degenerate nonlocal parabolic equation associated with the fractional Δ_p -Laplacian, *Dyn. Partial Differ. Equ.*, 14, 1, 47-77 (2017) · Zbl 06695569 · doi:10.4310/DPDE.2017.v14.n1.a4
- [19] Warma, M., The fractional Neumann and Robin type boundary conditions for the regional fractional Δ_p -Laplacian, *NoDEA Nonlinear Differential Equations Appl.*, 23, 1, 1-46 (2016) · Zbl 1338.35484 · doi:10.1007/s00030-016-0354-5
- [20] Grubb, G., Fractional Laplacians on domains, a development of Hörmander's theory of Δ_μ -transmission pseudodifferential operators, *Adv. Math.*, 268, 478-528 (2015) · Zbl 1318.47064 · doi:10.1016/j.aim.2014.09.018
- [21] Ghosh, T.; Salo, M.; Uhlmann, G., The Calderón problem for the fractional Schrödinger equation, *Analysis & PDE*, 13, 2, 455-475 (2020) · Zbl 1439.35530 · doi:10.2140/apde.2020.13.455
- [22] Valdinoci, E., From the long jump random walk to the fractional Laplacian, *Bol. Soc. Esp. Mat. Apl. SeMA*, 0, 49, 33-44 (2008) · Zbl 1242.60047
- [23] Arendt, W.; Elst, A. F. M. ter; Warma, M., Fractional powers of sectorial operators via the Dirichlet-to-Neumann operator, *Comm. Partial Differential Equations*, 43, 1, 1-24 (2018) · doi:10.1080/03605302.2017.1363229
- [24] Biccari, U.; Warma, M.; Zuazua, E., Addendum: Local elliptic regularity for the Dirichlet fractional Laplacian, *Adv. Nonlinear Stud.*, 17, 4, 837-839 (2017) · Zbl 06791441 · doi:10.1515/ans-2017-6020
- [25] Biccari, U.; Warma, M.; Zuazua, E., Local elliptic regularity for the Dirichlet fractional Laplacian, *Adv. Nonlinear Stud.*, 17, 2, 387-409 (2017) · Zbl 1360.35033 · doi:10.1515/ans-2017-0014
- [26] Ros-Oton, X.; Serra, J., The Dirichlet problem for the fractional Laplacian: regularity up to the boundary, *J. Math. Pures Appl.*, 101, 3, 275-302 (2014) · Zbl 1285.35020 · doi:10.1016/j.matpur.2013.06.003
- [27] Ros-Oton, X.; Serra, J., The extremal solution for the fractional Laplacian, *Calc. Var. Partial Differential Equations*, 50, 3-4, 723-750 (2014) · Zbl 1301.35204 · doi:10.1007/s00526-013-0653-1
- [28] Dipierro, S.; Ros-Oton, X.; Valdinoci, E., Nonlocal problems with Neumann boundary conditions, *Rev. Mat. Iberoam.*, 33, 2, 377-416 (2017) · Zbl 1371.35322 · doi:10.4171/RMI/942
- [29] Showalter, R. E., Partial differential equations of Sobolev-Galpern type, *Pacific J. Math.*, 31, 787-793 (1969) · Zbl 0185.19002 · doi:10.2140/pjm.1969.31.787
- [30] Showalter, R. E.; Ting, T. W., Pseudoparabolic partial differential equations, *SIAM J. Math. Anal.*, 1, 1-26 (1970) · Zbl 0199.42102 · doi:10.1137/0501001
- [31] Favini, A.; Yagi, A., *Degenerate Differential Equations in Banach Spaces* (1998), FL: CRC Press, FL · Zbl 0792.34059 · doi:10.1201/9781482276022

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.