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**The convergence of a numerical scheme for additive fractional stochastic delay equations with  $H > \frac{1}{2}$ .** (English) [Zbl 07431702](#)

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Summary: In this paper, we investigate the strong convergence of the exponential Euler method to stochastic delay differential equations with fractional Brownian motion (FSDDEs) of Hurst parameter  $H \in (\frac{1}{2}, 1)$ . We establish the strong convergence rate  $H$  of the method for FSDDEs to the exact solution. Also we justify our theoretical results with some numerical examples of these equations alongside insignificant step size.

**MSC:**

65-XX Numerical analysis

60-XX Probability theory and stochastic processes

**Keywords:**

stochastic delay differential equations; fractional Brownian motion; exponential Euler scheme; strong convergence

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