Recent advances in directional statistics. (English) [Zbl 07431351]
Test 30, No. 1, 1-58 (2021)

Summary: Mainstream statistical methodology is generally applicable to data observed in Euclidean space. There are, however, numerous contexts of considerable scientific interest in which the natural supports for the data under consideration are Riemannian manifolds like the unit circle, torus, sphere, and their extensions. Typically, such data can be represented using one or more directions, and directional statistics is the branch of statistics that deals with their analysis. In this paper, we provide a review of the many recent developments in the field since the publication of K. V. Mardia and P. E. Jupp [Directional statistics. Chichester: Wiley (2000; Zbl 0935.62065)], still the most comprehensive text on directional statistics. Many of those developments have been stimulated by interesting applications in fields as diverse as astronomy, medicine, genetics, neurology, space situational awareness, acoustics, image analysis, text mining, environmetrics, and machine learning. We begin by considering developments for the exploratory analysis of directional data before progressing to distributional models, general approaches to inference, hypothesis testing, regression, nonparametric curve estimation, methods for dimension reduction, classification and clustering, and the modelling of time series, spatial and spatio-temporal data. An overview of currently available software for analysing directional data is also provided, and potential future developments are discussed.

MSC:

62R30 Statistics on manifolds
62H11 Directional data; spatial statistics
62H30 Classification and discrimination; cluster analysis (statistical aspects)
62M10 Time series, auto-correlation, regression, etc. in statistics (GARCH)
62M30 Inference from spatial processes
62-02 Research exposition (monographs, survey articles) pertaining to statistics
62-04 Software, source code, etc. for problems pertaining to statistics

Keywords:
classification; clustering; dimension reduction; distributional models; exploratory data analysis; hypothesis tests; nonparametric methods; regression; serial dependence; software; spatial statistics

Software:
moveHMM; SiZer ; move; sphunif; CircOutlier; rotasym; rgl; DirStats; FCM4DD; rcosmo; BAMBI; VMF-SNE; bReeze; CircSIZER; circular; RiemBase; plot3D; CircStats; nprotreg; bpDir; movMF; geostats; sdetorus; Clustangles; PyCircStat; CircStat; CircSpaceTime; GeoPCA; shapes; CircNNTSR; season; FLightR; Directional; Mocapy ++; isocir; NHMSAR; R; depth

Full Text: DOI

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