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Genuine nonlinearity and its connection to the modified Korteweg-de Vries equation in phase dynamics. (English) [Zbl 1479.35748](#)

Nonlinearity 35, No. 1, 30-65 (2022).

MSC:

- 35Q53 KdV equations (Korteweg-de Vries equations)
- 35C20 Asymptotic expansions of solutions to PDEs
- 35L65 Hyperbolic conservation laws
- 78A60 Lasers, masers, optical bistability, nonlinear optics
- 35Q60 PDEs in connection with optics and electromagnetic theory
- 76B15 Water waves, gravity waves; dispersion and scattering, nonlinear interaction
- 76B55 Internal waves for incompressible inviscid fluids
- 76B70 Stratification effects in inviscid fluids
- 35Q35 PDEs in connection with fluid mechanics

Keywords:

Lagrangian dynamics; modulation; internal waves; optics; nonlinear waves; hyperbolic systems

Full Text: [DOI](#) [arXiv](#)

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