zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Dong, Guozhi; Hintermueller, Michael; Zhang, Ye

A class of second-order geometric quasilinear hyperbolic PDEs and their application in imaging. (English) Zbl 1478.35150

SIAM J. Imaging Sci. 14, No. 2, 645-688 (2021).

MSC:

35L72 Second-order quasilinear hyperbolic equations
35L80 Degenerate hyperbolic equations
49K20 Optimality conditions for problems involving partial differential equations
49J52 Nonsmooth analysis
65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

Keywords:

second-order quasilinear hyperbolic equation; geometric PDEs; total variation flow; mean curvature flow; level set; second-order dynamics; nonsmooth and nonconvex variational methods; image denoising; displacement error correction

Full Text: DOI arXiv

References:

[1] F. Alvarez, On the minimizing property of a second-order dissipative system in Hilbert spaces, SIAM J. Control Optimi., 38 (2000), pp. 1102-1119, https://doi.org/10.1137/S0363012998335802. · Zbl 0954.34053

[2] F. Alvarez, H. Attouch, J. Bolte, and P. Redont, A second-order gradient-like dissipative dynamical system with Hessian-driven damping. Application to optimization and mechanics, J. Math. Pures Appl., 81 (2002), pp. 747-779, https://doi.org/10.1016/S0021-7824(01)01253-3. · Zbl 1036.34072

[3] L. Ambrosio, N. Fusco, and D. Pallara, Functions of Bounded Variation and Free Discontinuity Problems, Oxford University Press, New York, 2000. · Zbl 0957.49001

[4] L. Ambrosio and H. Soner, Level set approach to mean curvature flow in arbitrary codimension, J. Differential Geom., 43 (1996), pp. 693-737, https://doi.org/10.4310/jdg/1214458529. · Zbl 0868.35046

[5] F. Andreu, C. Ballester, V. Caselles and J. M. Mazón, The Dirichlet problem for the total variation flow, J. Funct. Anal., 180 (2001), pp. 347-403, https://doi.org/10.1006/jfan.2000.3698. · Zbl 0973.35109

[6] F. Andreu-Vaillo, J. M. Mazón, and V. Caselles, Parabolic Quasilinear Equations Minimizing Linear Growth Functionals, Progr. Math. 223, Springer, Basel, 2004, https://doi.org/10.1007/978-3-0348-7928-6. · Zbl 1053.35002

[7] A. Ascanelli, Gevrey well posedness for a second order weakly hyperbolic equation with non-regular in time coefficients, Tsukuba J. Math., 30 (2006), pp. 415-431, https://doi.org/10.21099/tkbjm/1496165071. · Zbl 1133.35072

[8] H. Attouch and A. Cabot, Asymptotic stabilization of inertial gradient dynamics with time-dependent viscosity, J. Differential Equations, 263 (2017), pp. 5412-5458, https://doi.org/10.1016/j.jde.2017.06.024. · Zbl 1405.37092

[9] H. Attouch, A. Cabot, J. Peypouquet, and P. Redon, Fast convergence of inertial dynamics and algorithms with asymptotic vanishing viscosity, Math. Program., 168 (2017), pp. 123-175, https://doi.org/10.1007/s10107-016-0992-8.

[10] H. Attouch, Z. Chbanib, and R. Hassan, Combining fast inertial dynamics for convex optimization with Tikhonov regularization, J. Math. Anal. Appl., 457 (2018), pp. 1065-1094, https://doi.org/10.1016/j.jmaa.2016.12.017. · Zbl 1375.65080

[11] H. Attouch, X. Goudou, and P. Redont, The heavy ball with friction method. I. The continuous dynamical system, Communi. Contemp. Math., 2 (2000), pp. 1-34, https://doi.org/10.1142/S0219199700000025. · Zbl 0983.37016

[12] C. Ballester, V. Caselles, and M. Novaga, The total variation flow in \(\mathbb R\sp N\), J. Differential Equations, 184 (2002), pp. 475-525, https://doi.org/10.1006/jdeq.2001.4150. · Zbl 1036.35099

[13] G. Baravdish, O. Svensson, M. Gulliksson, and Y. Zhang, Damped second order flow applied to image denoising, IMA J. Appl. Math., 84 (2019), pp. 1082-1111, https://doi.org/10.1093/imamat/hxz027. · Zbl 1434.65163

[14] J. Barrett, H. Garcke, and R. Nürnberg, Numerical approximation of anisotropic geometric evolution equations in the plane, IMA J. Numer. Anal., 28 (2008), pp. 292-330, https://doi.org/10.1093/imanum/drm013. · Zbl 1145.65069

[15] S. Bartels, L. Diening, and R. Nochetto, Unconditional stability of semi-implicit discretizations of singular flows, SIAM J. Numer. Anal., 56 (2018), pp. 1896-1914, https://doi.org/10.1137/17M1159166. · Zbl 1394.65098

[16] M. Benyamin, J. Calder, G. Sundaramoorthi, and A. Yezzi, Accelerated variational PDEs for efficient solution of regularized

inversion problems, J. Math. Imaging Vision, 62 (2020), pp. 10â36, https://doi.org/10.1007/s10851-019-00910-2. · Zbl 1440.65124

[17] R. Bot, G. Dong, P. Elbau, and O. Scherzer, Convergence Rates of First and Higher Order Dynamics for Solving Linear Ill-Posed Problems, preprint, arXiv:1812.09343v1, 2018.

[18] H. Brezis, Functional Analysis, Soblev Spaces and Partial Differential Equations, 2nd ed., Universitext, Springer, 2010.

[19] L. Bungert, M. Burger, A. Chambolle, and M. Novaga, Nonlinear Spectral Decompositions by Gradient Flows of One-Homogeneous Functionals, preprint, arXiv:1901.06979v2, 2020. · Zbl 07365582

[20] M. Burger, G. Gilboa, M. Moeller, L. Eckardt, and D. Cremers, Spectral decompositions using one-homogeneous functionals, SIAM J. Imaging Sci., 9 (2016), pp. 1374-1408, https://doi.org/10.1137/15M1054687. · Zbl 1361.35123

[21] M. Burger, G. Gilboa, S. Osher, and J. Xu, Nonlinear inverse scale space methods, Commun. Math. Sci., 4 (2006), pp. 179-212, https://doi.org/euclid.cms/1145905942. · Zbl 1106.68117

[22] A. Cabot, H. Engler, and S. Gadat, On the long time behavior of second order differential equations with asymptotically small dissipation, Trans. Amer. Math. Soc., 361 (2009), pp. 5983-6017, https://doi.org/10.1090/S0002-9947-09-04785-0. · Zbl 1191.34078

[23] A. Chambolle and P. Lions, Image recovery via total variation minimization and related problems, Numer. Math., 76 (1997), pp. 167-188, https://doi.org/10.1007/s002110050258. · Zbl 0874.68299

[24] A. Chambolle and T. Pock, A first-order primal-dual algorithm for convex problems with applications to imaging, J. Math. Imaging Vision, 40 (2011), pp. 120-145, https://doi.org/10.1007/s10851-010-0251-1. · Zbl 1255.68217

[25] Y. G. Chen, Y. Giga, and S. Goto, Uniqueness and existence of viscosity solutions of generalized mean curvature flow equations, J. Differential Geom., 33 (1991), pp. 749-786, https://doi.org/10.4310/jdg/1214446564. · Zbl 0696.35087

[26] P. Cherrier and A. Milani, Decay estimates for quasi-linear evolution equations, Bull. Sci. Math., 135 (2011), pp. 33-58, https://doi.org/10.1016/j.bulsci.2010.04.004. · Zbl 1213.35095

[27] P. D'Ancona, Gevrey well-posedness of an abstract Cauchy problem of weakly hyperbolic type, Publ. Res. Inst. Math. Sci., 24 (1988), pp. 433-449, https://doi.org/10.2977/prims/1195175035. · Zbl 0706.35077

[28] K. Deckelnick, G. Dziuk, and C. Elliott, Computation of geometric partial differential equations and mean curvature flow, Acta Numer., 14 (2005), pp. 139â232, https://doi.org/10.1017/S0962492904000224. · Zbl 1113.65097

[29] P. A. Dionne, Sur les problémes de Cauchy hyperboliques bien posés, J. Anal. Math., 10 (1962), pp. 1-90, https://doi.org/10.1007/BF02790303. · Zbl 0112.32301

[30] G. Dong, A. Patrone, O. Scherzer, and O. Öktem, Infinite dimensional optimization models and PDEs for dejittering, in Scale Space and Variational Methods in Computer Vision, Lecture Notes in Comput. Sci. 9087, Springer, New York, 2015, pp. 678-689, https://doi.org/10.1007/978-3-319-18461-6_54. · Zbl 1444.68282

[31] G. Dong and O. Scherzer, Nonlinear flows for displacement correction and applications in tomography, in Scale Space and Variational Methods in Computer Vision, F. Lauze, Y. Dong, and A. Dahl, eds., Lecture Notes in Comput. Sci. 10302, Springer, New York, 2017, pp. 283-294, https://doi.org/10.1007/978-3-319-58771-4_23.

[32] P. Elbau, M. Grasmair, F. Lenzen, and O. Scherzer, Evolution by non-convex functionals, Numer. Funct. Anal. Optim., 31 (2010), pp. 489-517, https://doi.org/10.1080/01630563.2010.485853. · Zbl 1202.35008

[33] H. Engl, M. Hanke, and A. Neubauer, Regularization of Inverse Problems, Kluwer, Dordrecht, 1996. · Zbl 0859.65054

[34] L. Evans, Partial Differential Equations, 2nd ed., Grad. Stud. Math. 19, AMS, Providence, RT, 2010. · Zbl 1194.35001

[35] L. Evans and J. Spruck, Motion of level sets by mean curvature. I, J. Differential Geom., 33 (1991), pp. 635-681, https://doi.org/10.4310/jdg/121444 · Zbl 0726.53029

[36] S. Gerschgorin, Über die abgrenzung der eigenwerte einer matrix, Bull. Acad. Sci. URSS, 6 (1931), pp. 749-754. · Zbl 0003.00102

[37] G. Gilboa, A total variation spectral framework for scale and texture analysis, SIAM J. Imaging Sci., 7 (2014), pp. 1937-1961, https://doi.org/10.1137/130930704. · Zbl 1361.94014

[38] E. Ginder and K. Svadlenka, Wave-type threshold dynamics and the hyperbolic mean curvature flow, Jpn. J. Ind. Appl. Math., 33 (2016), pp. 501-523, https://doi.org/10.1007/s13160-016-0221-0. · Zbl 1354.65209

[39] R. Gonzalez and R. Woods, Digital Image Processing, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 2002.

[40] C. He, D. Kong, and K. Liu, Hyperbolic mean curvature flow, J. Differential Equations, 246 (2009), pp. 373-390, https://doi.org/10.1016/j.jde.2008.0 · Zbl 1159.53024

[41] M. Hintermüller, C. Rautenberg, and J. Hahn, Functional-analytic and numerical issues in splitting methods for total variation-based image reconstruction, Inverse Porblems, 30 (2014), 055014, https://doi.org/10.1088/0266-5611/30/5/055014. · Zbl 1293.94015

[42] F. John, Delayed singularity formation in solution of nonlinear wave equations in higher dimensions, Commun. Pure Appl. Math., 29 (1976), pp. 649-682, https://doi.org/10.1002/cpa.3160290608. · Zbl 0332.35044

[43] S. Klainerman, Global existence for nonlinear wave equations, Commun. Pure Appl. Math., 33 (1980), pp. 43-101, https://doi.org/10.1002/cpa.3160 · Zbl 0405.35056

[44] P. Lax, Hyperbolic Partial Differential Equations, Courant Lect. Note Math. 14, Courant Institute of Mathematical Sciences, New York, 2006. · Zbl 1113.35002

[45] P. LeFloch and K. Smoczyk, The hyperbolic mean curvature flow, J. Math. Pures Appl., 90 (2008), pp. 591-614, https://doi.org/10.1016/j.matpur.20 · Zbl 1159.53025

[46] F. Lenzen and O. Scherzer, Partial differential equations for zooming, deinterlacing and dejittering, Internat. J. Computer

Vision, 92 (2011), pp. 162-176, https://doi.org/10.1007/s11263-010-0326-x. · Zbl 1235.68312

[47] R. Malladi and J. Sethian, Image processing: Flows under min/max curvature and mean curvature, Graphical Models Image Processing, 58 (1996), pp. 127-141, https://doi.org/10.1006/gmip.1996.0011.

[48] A. Matsumura, Global existence and asymptotics of the solutions of the second-order quasilinear hyperbolic equations with the first-order dissipation, Publ. Res. Inst. Math. Sci., 13 (1977), pp. 349-379. · Zbl 0371.35030

[49] Y. Nesterov, Introductory Lectures on Convex Optimization: A Basic Course, Appl. Optim. 87, Springer, New York, 2004, https://doi.org/10.1007/978-1-4419-8853-9. · Zbl 1086.90045

[50] S. Osher and J. Sethian, Fronts propagating with curvature dependent speed: Algorithms based on Hamilton-Jacobi formulations, J. Comput. Phys., 79 (1988), pp. 12-49, https://doi.org/10.1016/0021-9991(88)90002-2. · Zbl 0659.65132

[51] L. I. Rudin, S. Osher, and E. Fatemi, Nonlinear total variation based noise removal algorithms, Phys. D, 60 (1992), pp. 259-268, https://doi.org/10.1016/0167-2789(92)90242-F. · Zbl 0780.49028

[52] O. Scherzer, M. Grasmair, H. Grossauer, M. Haltmeier, and F. Lenzen, Variational Methods in Imaging, Appl. Math. Sci. 167, Springer, New York, 2009, https://doi.org/10.1007/978-0-387-69277-7. · Zbl 1177.68245

[53] W. Su, S. Boyd, and E. Candes, A differential equation for modeling Nesterov's accelerated gradient method: Theory and insights, J. Mach. Learn. Res., 17 (2016), pp. 5312-5354. · Zbl 1391.90667

[54] S. Yau, Review of geometry and analysis, Asian J. Math., 4 (2000), pp. 235â278, https://doi.org/10.4310/AJM.2000.v4.n1.a16. · Zbl 1031.53004

[55] Y. Zhang and B. Hofmann, On the second order asymptotical regularization of linear ill-posed inverse problems, Appl. Anal., 99 (2020), pp. 1000-1025, https://doi.org/10.1080/00036811.2018.1517412. · Zbl 1443.47014

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.