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Claeys, Xavier; **Giacomel, Lorenzo**; **Hiptmair, Ralf**; **Urzúa-Torres, Carolina**

Quotient-space boundary element methods for scattering at complex screens. (English)

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Summary: A complex screen is an arrangement of panels that may not be even locally orientable because of junction lines. A comprehensive trace space framework for first-kind variational boundary integral equations on complex screens has been established in [*X. Claeys* and *R. Hiptmair*, Integral Equations Oper. Theory 77, No. 2, 167–197 (2013; Zbl 1292.45001)] for the Helmholtz equation, and in [*X. Claeys* and *R. Hiptmair*, Integral Equations Oper. Theory 84, No. 1, 33–68 (2016; Zbl 1337.78008)] for Maxwell's equations in frequency domain. The gist is a *quotient space perspective* that allows to make sense of jumps of traces as factor spaces of multi-trace spaces modulo single-trace spaces without relying on orientation. This paves the way for formulating first-kind boundary integral equations in weak form posed on energy trace spaces. In this article we extend that idea to the Galerkin boundary element (BE) discretization of first-kind boundary integral equations. Instead of trying to approximate jumps directly, the new quotient space boundary element method employs a Galerkin BE approach in multi-trace boundary element spaces. This spawns discrete boundary integral equations with large null spaces comprised of single-trace functions. Yet, since the right-hand-sides of the linear systems of equations are consistent, Krylov subspace iterative solvers like GMRES are not affected by the presence of a kernel and still converge to a solution. This is strikingly confirmed by numerical tests.

MSC:

65N38 Boundary element methods for boundary value problems involving PDEs
65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
78M15 Boundary element methods applied to problems in optics and electromagnetic theory
78M10 Finite element, Galerkin and related methods applied to problems in optics and electromagnetic theory
76M15 Boundary element methods applied to problems in fluid mechanics
76M10 Finite element methods applied to problems in fluid mechanics
78A45 Diffraction, scattering
76Q05 Hydro- and aero-acoustics
35R09 Integro-partial differential equations
45K05 Integro-partial differential equations
45A05 Linear integral equations
65R20 Numerical methods for integral equations
65F10 Iterative numerical methods for linear systems

Keywords:

complex screens; Galerkin boundary element method; quotient space boundary element method

Software:

BETL; MINRES

Full Text: DOI

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