Dębski, Wojciech; **Kawamura, Kazuhiro**; **Tuncali, Murat**; **Tymchatyn, Edward D.**
Inverse systems with simplicial bonding maps and cell structures. (English) Zbl 1483.54006
Topology Appl. 304, Article ID 107790, 14 p. (2021).

The main result of the paper is: given a topologically complete space X and a "defining family of closed covers" $\mathcal{A}$ of X (i.e., it satisfies some local refinement condition and completeness condition), to construct an inverse system $\mathbf{F}_{\mathcal{A}} = (F_\lambda, \pi^\mu_\lambda, \Lambda)$ of simplicial complexes and simplicial bonding maps, and an inverse system $\mathbf{N}_{\mathcal{A}} = (N_\lambda, \pi^\mu_\lambda, \Lambda)$ such that the limit spaces $F_\infty = N_\infty$, where N_λ is a subcomplex of F_λ which is almost the same as a cover, and to obtain a proper map $\pi : N_\infty \to X$, and a continuous map $p : X \to N_\infty$ so that $\pi \circ p = \mathrm{id}_X$ and the two maps $p \circ \pi$ and id_{N_∞} are homotopic (so N_∞ is homotopy equivalent to X). The construction of the inverse system is based on a modification of the theorem by *S. Mardešić* [Fundam. Math. 114, 53–78 (1981; Zbl 0411.54019)] that every topological space X admits a polyhedral resolution.

The authors then show that if X is a compact Hausdorff space and $\mathcal{A}$ is a family of locally finite, normal, closed covers of X satisfying the local refinement condition, then the inverse systems $\mathbf{F}_{\mathcal{A}}$ and $\mathbf{N}_{\mathcal{A}}$ are HPol-expansions of N_∞ and hence of X, where an expansion is in the shape theoretical sense.

They also show that the restricted inverse system $\mathbf{F}^{(0)} = (F^{(0)}_\lambda, \pi^\mu_\lambda, \Lambda)$ is a cell structure in the sense of [*W. Dębski* and *E. D. Tymchatyn*, Topology Appl. 239, 293–307 (2018; Zbl 1390.54014)] representing a space canonically homeomorphic to X.

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MSC:

54C05 Continuous maps
54B35 Spectra in general topology
54D30 Compactness
54E15 Uniform structures and generalizations
78A70 Biological applications of optics and electromagnetic theory

Keywords:

polyhedral inverse system; flag complexes; cell structures; discrete approximation of spaces; shape theory

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