

Cervera, Miguel; Wu, Jian-Ying; Chiumenti, Michele; Kim, Sungchul
Strain localization analysis of Hill's orthotropic elastoplasticity: analytical results and numerical verification. (English) [Zbl 1481.74084](#)
Comput. Mech. 65, No. 2, 533-554 (2020).

Summary: In this work the strain localization analysis of Hill's orthotropic plasticity is addressed. In particular, the localization condition derived from the boundedness of stress rates together with Maxwell's kinematics is employed. Similarly to isotropic plasticity considered in our previous work, the plastic flow components on the discontinuity surface vanish upon strain localization. The resulting localization angles in orthotropic plastic materials are independent from the elastic constants, but rather, they depend on the material parameters involved in the plastic flow in the material axes. Application of the above localization condition to Hill's orthotropic plasticity in 2-D plane stress and plane strain conditions yields closed-form solutions of the localization angles. It is found that the two discontinuity lines in plane strain conditions are always perpendicular to each other, and for the states of no shear stresses, the localization angle depends only on the tilt angle of the material axes with respect to the global ones. The analytical results are then validated by *independent* numerical simulations. The *B*-bar finite element is employed to deal with the incompressibility due to the purely isochoric plastic flow. For a strip under vertical stretching in plane stress and plane strain as well as Prandtl's problem of indentation by a flat rigid die in plane strain, numerical results are presented for both isotropic and orthotropic plasticity models with or without tilt angle. The influence of various parameters is studied. In all cases, the critical angles predicted from the localization condition coincide with the numerical results, giving compelling supports to the analytical prognoses.

MSC:

- 74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)
- 74E10 Anisotropy in solid mechanics
- 74S05 Finite element methods applied to problems in solid mechanics

Keywords:

bifurcation; strain localization; orthotropic elastoplasticity; incompressibility; finite element simulation; Prandtl problem

Full Text: [DOI](#)

References:

- [1] Prandtl L (1920) Über die häete plastischer körper. Nachr. Ges. Wissensch, Göttingen, math. phys. Klasse, pp 74-85
- [2] Hencky, H., Über einige statisch bestimmte fälle des gleichgewichts in plastischen körpern, Z Angew Math Mech, 3, 241-251 (1923) · [Zbl 49.0596.01](#)
- [3] Hencky, H., Zur theorie plastischer deformationen und der hierdurch im material hervorgerufenen nachspannungen, Z Angew Math Mech, 4, 323-334 (1924) · [Zbl 50.0546.03](#)
- [4] Mandel J (1942) Equilibre par trasches planes des solides à la limite d'écoulement. PhD thesis, Thèse, Paris
- [5] Hill, R., The mathematical theory of plasticity (1950), New York: Oxford University Press, New York · [Zbl 0041.10802](#)
- [6] Hill, R., On discontinuous plastic states, with special reference to localized necking in thin sheets, J Mech Phys Solids, 1, 19-30 (1952)
- [7] Hill, R., General theory of uniqueness and stability of elasto-plastic solids, J Mech Phys Solids, 6, 236-249 (1958) · [Zbl 0091.40301](#)
- [8] Hill, R., Acceleration waves in solids, J Mech Phys Solids, 10, 1-16 (1962) · [Zbl 0111.37701](#)
- [9] Thomas, Ty, Plastic flow and fracture of solids (1961), New York: Academic Press, New York
- [10] Rice, Jr, A path independent integral and the approximate analysis of strain cncentrations by notches and cracks, J Appl Mech Trans ASME, 35, 379-386 (1968)
- [11] Rudnicki, Jw; Rice, Jr, Conditions of the localization of deformation in pressure-sensitive dilatant material, J Mech Phys

Solids, 23, 371-394 (1975)

- [12] Rice, Jr; Rudnicki, Jw, A note on some features of the theory of localization of deformation, *Int J Solids Struct*, 16, 597-605 (1980) · [Zbl 0433.73032](#)
- [13] Runesson, K.; Ottosen, Ns; Peric, D., Discontinuous bifurcations of elastic-plastic solutions at plane stress and plane strain, *Int J Plast*, 7, 99-121 (1991) · [Zbl 0761.73035](#)
- [14] Simó, Jc; Oliver, J.; Armero, F., An analysis of strong discontinuities induced by strain-softening in rate-independent inelastic solids, *Comput Mech*, 12, 277-296 (1993) · [Zbl 0783.73024](#)
- [15] Oliver J (1996) Modeling strong discontinuities in solid mechanics via strain softening constitutive equations. Part I: fundamentals; Part II: numerical simulation. *Int J Numer Methods Eng* 39:3575-3600; 3601-3623 · [Zbl 0888.73018](#)
- [16] Oliver, J.; Cervera, M.; Manzoli, O., Strong discontinuities and continuum plasticity models: the strong discontinuity approach, *Int J Plast*, 15, 319-351 (1999) · [Zbl 1057.74512](#)
- [17] Oliver, J., On the discrete constitutive models induced by strong discontinuity kinematics and continuum constitutive equations, *Int J Solids Struct*, 37, 7207-7229 (2000) · [Zbl 0994.74004](#)
- [18] Cervera, M.; Chiumenti, M.; Di Capua, D., Benchmarking on bifurcation and localization in (j_2) plasticity for plane stress and plane strain conditions, *Comput Methods Appl Mech Eng*, 241244, 206224 (2012) · [Zbl 1353.74033](#)
- [19] Oliver, J.; Huespe, Ae; Dias, If, Strain localization, strong discontinuities and material fracture: matches and mismatches, *Comput Methods Appl Mech Eng*, 241, 323-336 (2006) · [Zbl 1353.74009](#)
- [20] Wu, Jy; Cervera, M., On the equivalence between traction- and stress-based approaches for the modeling of localized failure in solids, *J Mech Phys Solids*, 82, 137-163 (2015)
- [21] Wu, Jy; Cervera, M., A thermodynamically consistent plastic-damage framework for localized failure in quasi-brittle solids: material model and strain localization analysis, *Int J Solids Struct*, 88-89, 227-247 (2016)
- [22] Wu, Jy; Cervera, M., Strain localization of elastic-damaging frictional-cohesive materials: analytical results and numerical verification, *Materials*, 10, 434 (2017)
- [23] Cervera, M.; Chiumenti, M.; Benedetti, L.; Codina, R., Mixedstabilized finite element methods in nonlinear solid mechanics. Part III: compressible and incompressible plasticity, *Comput Methods Appl Mech Eng*, 285, 752-775 (2015) · [Zbl 1423.74149](#)
- [24] Hoffman, O., The brittle strength of orthotropic materials, *J Comp Mater*, 1, 200-206 (1967)
- [25] Tsai, Sw; Wu, Em, A general theory of strength for anisotropic materials, *J Comp Mater*, 5, 58-80 (1971)
- [26] Oller, S.; Car, E.; Lubliner, J., Definition of a general implicit orthotropic yield criterion, *Comput Methods Appl Mech Eng*, 192, 895-912 (2003) · [Zbl 1112.74325](#)
- [27] Li, M.; Füssl, J.; Lukacevic, M.; Eberhardsteiner, J., A numerical upper bound formulation with sensibly-arranged velocity discontinuities and orthotropic material strength behavior, *J Theor Appl Mech*, 56, 2, 417-433 (2018)
- [28] Rots, Jg; Nauta, P.; Kusters, Gma; Blaauwendraad, J., Smeared crack approach and fracture localization in concrete, *Heron*, 30, 1-47 (1985)
- [29] Jirásek, M.; Zimmermann, T., Analysis of rotating crack model, *J Eng Mech ASCE*, 124, 8, 842-851 (1998)
- [30] Cervera, M., A smeared-embedded mesh-corrected damage model for tensile cracking, *Int J Numer Methods Eng*, 76, 1930-1954 (2008) · [Zbl 1195.74153](#)
- [31] Cervera, M., An orthotropic mesh corrected crack model, *Comput Methods Appl Mech Eng*, 197, 1603-1619 (2008) · [Zbl 1194.74295](#)
- [32] Wu JY, Cervera M (2014) Strain localization and failure mechanics for elastoplastic damage solids. Monograph CIMNE M147, Barcelona, Spain
- [33] Hill, R., A theory of the yielding and plastic flow of anisotropic metals, *Proc R Soc A*, 193, 281-297 (1948) · [Zbl 0032.08805](#)
- [34] Simó, Jc; Hughes, Tjr, *Computational inelasticity* (1998), New York: Springer, New York
- [35] Cervera, M.; Chiumenti, M., Size effect and localization in (j_2) plasticity, *Int J Solids Struct*, 46, 3301-3312 (2009) · [Zbl 1167.74341](#)
- [36] Felippa, Ca; Oñate, E., Stress, strain and energy splittings for anisotropic solids under volumetric constraints, *Comput Struct*, 81, 13, 1343-1357 (2003)
- [37] Hughes, Tjr, Generalization of selective intertegration procedures to anisotropic and nonlinear media, *Int J Numer Methods Eng*, 15, 9, 1413-1418 (1980) · [Zbl 0437.73053](#)
- [38] Hughes, Trj, *The finite element method. Linear static and dynamic finite element analysis* (2000), Mineola: Dover Publications Inc., Mineola

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.