Choi, Young-Pil; **Jeong, In-Jee**

On well-posedness and singularity formation for the Euler-Riesz system. (English)

Zbl 1477.35143
J. Differ. Equations 306, 296-332 (2022).

Summary: In this paper, we investigate the initial value problem for the Euler-Riesz system, where the interaction forcing is given by $\nabla(-\Delta)^s\rho$ for some $-1 < s < 0$, with $s = -1$ corresponding to the classical Euler-Poisson system. We develop a functional framework to establish local-in-time existence and uniqueness of classical solutions for the Euler-Riesz system. In this framework, the fluid density could decay fast at infinity, and the Euler-Poisson system can be covered as a special case. Moreover, we prove local well-posedness for the pressureless Euler-Riesz system when the potential is repulsive, by observing hyperbolic nature of the system. Finally, we present sufficient conditions on the finite-time blowup of classical solutions for the isentropic/isothermal Euler-Riesz system with either attractive or repulsive interaction forces. The proof, which is based on estimates of several physical quantities, establishes finite-time blowup for a large class of initial data; in particular, it is not required that the density is of compact support.

MSC:

35Q31 Euler equations
35Q35 PDEs in connection with fluid mechanics
35B44 Blow-up in context of PDEs
35A01 Existence problems for PDEs: global existence, local existence, non-existence
35A02 Uniqueness problems for PDEs: global uniqueness, local uniqueness, non-uniqueness
35A09 Classical solutions to PDEs

Keywords:

Euler-Riesz system; finite-time blowup of classical solutions

References:

[1] Bae, Junsik; Choi, Junho; Kwon, Bongsuk, Formation of singularities in cold ion dynamics · Zbl 1406.35256

[2] Brauer, Uwe; Karp, Lavi, Local existence of solutions to the Euler-Poisson system, including densities without compact support, J. Differ. Equ., 264, 2, 755-785 (2018), MR 3720829 · Zbl 1378.35302

[3] Brenier, Y.; Gangbo, W.; Savaré, G.; Westdickenberg, M., Sticky particle dynamics with interactions, J. Math. Pures Appl. (9), 99, 5, 577-617 (2013), MR 3039208 · Zbl 1282.35236

[4] Brenier, Yann; Grenier, Emmanuel, Sticky particles and scalar conservation laws, SIAM J. Numer. Anal., 35, 6, 2317-2328 (1998), MR 1655848 · Zbl 0924.35080

[5] Carrillo, José A.; Choi, Young-Pil, Mean-field limits: from particle descriptions to macroscopic equations, Arch. Ration. Mech. Anal., 241, 1529-1573 (2021) · Zbl 1478.35170

[6] Carrillo, José A.; Choi, Young-Pil, Quantitative error estimates for the large friction limit of Vlasov equation with nonlocal forces, Ann. Inst. Henri Poincaré, Anal. Non Linéaire, 37, 4, 925-954 (2020), MR 4104830 · Zbl 1440.35322

[7] Carrillo, José A.; Choi, Young-Pil; Hauray, Maxime; Salem, Samir, Mean-field limit for collective behavior models with sharp sensitivity regions, J. Eur. Math. Soc., 21, 1, 121-161 (2019), MR 3880206 · Zbl 1404.92222

[8] Carrillo, José A.; Choi, Young-Pil; Jung, Jinwwok, Quantifying the hydrodynamic limit of Vlasov-type equations with alignment and nonlocal forces, Math. Models Methods Appl. Sci., 31, 02, 327-408 (2021), MR 4227888 · Zbl 1483.35267

[9] Carrillo, José A.; Choi, Young-Pil; Salem, Samir, Propagation of chaos for the Vlasov-Poisson-Fokker-Planck equation with a polynomial cut-off, Commun. Contemp. Math., 21, 4, Article 1850039 pp. (2019), MR 3957154 · Zbl 1417.35201

[10] Carrillo, José A.; Choi, Young-Pil; Zatorska, Ewelina, On the pressureless damped Euler-Poisson equations with quadratic confinement: critical thresholds and large-time behavior, Math. Models Methods Appl. Sci., 26, 12, 2311-2340 (2016), MR 3564592 · Zbl 1349.35378

[11] Chae, Dongho; Constantin, Peter; Córdoba, Diego; Gancedo, Francisco; Wu, Jiahong, Generalized surface quasi-geostrophic equations with singular velocities, Commun. Pure Appl. Math., 65, 8, 1037-1066 (2012), MR 2928091 · Zbl 1244.35108

[12] Chae, Dongho; Ha, Seung-Yeal, On the formation of shocks to the compressible Euler equations, Commun. Math. Sci., 7, 3, 627-634 (2009), MR 2569026 · Zbl 1183.35225

[13] Chae, Dongho; Tadmor, Eitan, On the finite time blow-up of the Euler-Poisson equations in \(\mathbb{R}^n\), Commun. Math. Sci., 6, 3, 785-789 (2008), MR 2455476 · Zbl 1157.35086

[14] Chen, Gui-Qiang, Euler equations and related hyperbolic conservation laws, (Evolutionary Equations, vol. II. Evolutionary Equations, vol. II, Handb. Differ. Equ. (2005), Elsevier/North-Holland: Elsevier/North-Holland Amsterdam), 1-104, MR 2182827 · Zbl 1092.35062

[15] E, Weinan; Rykov, Yu. G.; Sinai, Ya. G., Generalized variational principles, global weak solutions and behavior with random initial data for systems of conservation laws arising in adhesion particle dynamics, Commun. Math. Phys., 177, 2, 349-380 (1996), MR 1384139 · Zbl 0852.35097

[16] Engelberg, Shlomo, Formation of singularities in the Euler and Euler-Poisson equations, Physica D, 98, 1, 67-74 (1996), MR 1416291 · Zbl 0885.35086

[17] Engelberg, Shlomo; Liu, Hailiang; Tadmor, Eitan, Critical Thresholds in Euler-Poisson Equations, vol. 50 (2001), Dedicated to Professors Ciprian Foias and Roger Temam (Bloomington, IN, 2000), pp. 109-157. MR 1855666 · Zbl 0989.35110

[18] Figalli, Alessio; Kang, Moon-Jin, A rigorous derivation from the kinetic Cucker-Smale model to the pressureless Euler system with nonlocal alignment, Anal. PDE, 12, 3, 843-866 (2019), MR 3864212 · Zbl 1405.35206

[19] Glassey, Robert T., The Cauchy Problem in Kinetic Theory (1996), Society for Industrial and Applied Mathematics (SIAM): Society for Industrial and Applied Mathematics (SIAM) Philadelphia, PA, MR 1379589 · Zbl 0858.76001

[20] Guo, Yan, Smooth irrotational flows in the large to the Euler-Poisson system in \(\mathbf{R}^{3 + 1} \), Commun. Math. Phys., 195, 2, 249-265 (1998), MR 1637856 · Zbl 0929.35112

[21] Guo, Yan; Pausader, Benoit, Global smooth ion dynamics in the Euler-Poisson system, Commun. Math. Phys., 303, 1, 89-125 (2011), MR 2775116 · Zbl 1220.35129

[22] Hauray, Maxime, Mean field limit for the one dimensional Vlasov-Poisson equation, (Séminaire Laurent Schwartz—Équations aux Dérivées Partielles et Applications. Séminaire Laurent Schwartz—Équations aux Dérivées Partielles et Applications, Année 2012-2013, Sémin. Équ. Dériv. Partielles (2014), École Polytech.: École Polytech. Palaiseau), Exp. No. XXI, 16. MR 3381008 · Zbl 1317.76032

[23] Hauray, Maxime; Jabin, Pierre-Emmanuel, N-particles approximation of the Vlasov equations with singular potential, Arch. Ration. Mech. Anal., 183, 3, 489-524 (2007), MR 2278413 · Zbl 1107.76066

[24] Hauray, Maxime; Jabin, Pierre-Emmanuel, Particle approximation of Vlasov equations with singular forces: propagation of chaos, Ann. Sci. Éc. Norm. Supér. (4), 48, 4, 891-940 (2015), MR 3377068 · Zbl 1329.35309

[25] Hauray, Maxime; Salem, Samir, Propagation of chaos for the Vlasov-Poisson-Fokker-Planck system in 1D, Kinet. Relat. Models, 12, 2, 269-302 (2019), MR 3918268 · Zbl 1420.35412

[26] Ionescu, Alexandru D.; Pausader, Benoit, The Euler-Poisson system in 2D: global stability of the constant equilibrium solution, Int. Math. Res. Not., 4, 761-826 (2013), MR 3024265 · Zbl 1320.35270

[27] Illner, R.; Victory, H. D.; Dukes, P.; Bobylev, A. V., On Vlasov-Manev equations. II. Local existence and uniqueness, J. Stat. Phys., 91, 3-4, 625-654 (1998), MR 1632718 · Zbl 0938.45008

[28] Jabin, Pierre-Emmanuel; Wang, Zhenfu, Quantitative estimates of propagation of chaos for stochastic systems with \(W^{-1 , \infty}\) kernels, Invent. Math., 214, 1, 523-591 (2018), MR 3858403 · Zbl 1402.35208

[29] Jeong, In-Jee; Kang, Kyungkeun, Well-posedness and singularity formation for inviscid Keller-Segel system of consumption type

[30] Jang, Juhi; Li, Dong; Zhang, Xiaoyi, Smooth global solutions for the two-dimensional Euler Poisson system, Forum Math., 26, 3, 645-701 (2014), MR 3200346 · Zbl 1298.35148

[31] Kang, Moon-Jin, From the Vlasov-Poisson equation with strong local alignment to the pressureless Euler-Poisson system, Appl. Math. Lett., 79, 85-91 (2018), MR 3748615 · Zbl 1462.35397

[32] Karper, Trygve K.; Mellet, Antoine; Trivisa, Konstantina, Hydrodynamic limit of the kinetic Cucker-Smale flocking model, Math. Models Methods Appl. Sci., 25, 1, 131-163 (2015), MR 3277287 · Zbl 1309.35180

[33] Kato, Tosio, Remarks on the Euler and Navier-Stokes equations in \(\mathbf{R}^2\), (Nonlinear Functional Analysis and Its Applications, Part 2. Nonlinear Functional Analysis and Its Applications, Part 2, Berkeley, Calif., 1983. Nonlinear Functional Analysis and Its Applications, Part 2. Nonlinear Functional Analysis and Its Applications, Part 2, Berkeley, Calif., 1983, Proc. Sympos. Pure Math., vol. 45 (1986), Amer. Math. Soc.: Amer. Math. Soc. Providence, RI), 1-7, MR 843590

[34] Kukavica, Igor; Vicol, Vlad; Wang, Fei, On the ill-posedness of active scalar equations with odd singular kernels, (New Trends in Differential Equations, Control Theory and Optimization (2016), World Sci. Publ.: World Sci. Publ. Hackensack, NJ), 185-200, MR 3587427 · Zbl 1348.35303

[35] Lazarovici, Dustin; Pickl, Peter, A mean field limit for the Vlasov-Poisson system, Arch. Ration. Mech. Anal., 225, 3, 1201-1231 (2017), MR 3667287 · Zbl 1375.35556

[36] Li, Yachun; Pan, Ronghua; Zhu, Shengguo, On classical solutions for viscous polytropic fluids with degenerate viscosities and vacuum, Arch. Ration. Mech. Anal., 234, 3, 1281-1334 (2019), MR 4011697 · Zbl 1428.35379

[37] Li, Dong; Wu, Yifei, The Cauchy problem for the two dimensional Euler-Poisson system, J. Eur. Math. Soc., 16, 10, 2211-2266 (2014), MR 3274788 · Zbl 1308.35220

[38] Natile, Luca; Savaré, Giuseppe, A Wasserstein approach to the one-dimensional sticky particle system, SIAM J. Math. Anal., 41, 4, 1340-1365 (2009), MR 2540269 · Zbl 1203.35170

[39] Nguyen, Truyen; Tudorascu, Adrian, Pressureless Euler/Euler-Poisson systems via adhesion dynamics and scalar conservation laws, SIAM J. Math. Anal., 40, 2, 754-775 (2008), MR 2438785 · Zbl 1171.35432

[40] Perthame, Benoît, Mathematical tools for kinetic equations, Bull. Am. Math. Soc., 41, 2, 205-244 (2004), MR 2043752 · Zbl 1151.82351

[41] Poupaud, F.; Rascle, M.; Vila, J.-P., Global solutions to the isothermal Euler-Poisson system with arbitrarily large data, J. Differ. Equ., 123, 1, 93-121 (1995), MR 1359913 · Zbl 0845.35123

[42] Serfaty, Sylvia, Mean field limit for Coulomb-type flows, Duke Math. J., 169, 15, 2887-2935 (2020), MR 4158670 · Zbl 1475.35341

[43] Sideris, Thomas C., Formation of singularities in three-dimensional compressible fluids, Commun. Math. Phys., 101, 4, 475-485 (1985), MR 815196 · Zbl 0606.76088

[44] Wang, Yuexun, Formation of singularities to the Euler-Poisson equations, Nonlinear Anal., 109, 136-147 (2014), MR 3247298 · Zbl 1301.35171

[45] Xin, Zhouping, Blowup of smooth solutions to the compressible Navier-Stokes equation with compact density, Commun. Pure Appl. Math., 51, 3, 229-240 (1998), MR 1488513 · Zbl 0937.35134