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Global stability of traveling waves for nonlocal time-delayed degenerate diffusion equation.

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Summary: This paper is concerned with a class of nonlocal reaction-diffusion equations with time-delay and degenerate diffusion. Affected by the degeneracy of diffusion, it is proved that, the Cauchy problem of the equation possesses the Hölder-continuous solution. Furthermore, the non-critical traveling waves are proved to be globally L^1 -stable, which is the first frame work on L^1 -wavefront-stability for the degenerate diffusion equations. The time-exponential convergence rate is also derived. The adopted approach for the proof is the technical L^1 -weighted energy estimates combining the compactness analysis, but with some new development.

MSC:

- [35B40](#) Asymptotic behavior of solutions to PDEs
- [35C07](#) Traveling wave solutions
- [35K15](#) Initial value problems for second-order parabolic equations
- [35K65](#) Degenerate parabolic equations
- [35K59](#) Quasilinear parabolic equations
- [35R09](#) Integro-partial differential equations
- [35R10](#) Partial functional-differential equations

Keywords:

degenerate reaction-diffusion equation; time-delay; traveling wave; stability; Hölder continuity

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