Kim, Yong-Cheol

Nonlocal Harnack inequalities for nonlocal Schrödinger operators with A_1-Muckenhoupt potentials. (English) Zbl 1478.35057

J. Math. Anal. Appl. 507, No. 1, Article ID 125746, 27 p. (2022).

In this paper, by applying the De Giorgi-Nash-Moser theory, the author obtains nonlocal Harnack inequalities for (locally nonnegative in Ω) weak solutions of the nonlocal Schrödinger equations

$$\begin{aligned} L_K u + Vu &= 0 \ \text{ in } \Omega, \\ u &= g \ \text{ in } \mathbb{R}^n \setminus \Omega \end{aligned}$$

where $V = V_+ - V_-$ with $V_- \in L^1_{\text{loc}}(\mathbb{R}^n)$ and $V_+ \in L^q_{\text{loc}}(\mathbb{R}^n)$, with $q > n$ and such that the potentials V, V_+, V_- belong to suitable A_1-Muckenhoupt classes. Noteworthy, this result implies the classical Harnack inequalities for globally nonnegative weak solutions of the equations. Nonlocal weak Harnack inequalities for weak supersolutions are a straightforwad consequence of the above results that are still working for any nonnegative potential in $L^q_{\text{loc}}(\mathbb{R}^n)$.

Reviewer: Vincenzo Vespri (Firenze)

MSC:

35B45 A priori estimates in context of PDEs
35B65 Smoothness and regularity of solutions to PDEs
35J10 Schrödinger operator, Schrödinger equation
35J15 Second-order elliptic equations
35R09 Integro-partial differential equations

Keywords:

nonlocal Harnack inequalities; nonlocal Schrödinger operators; A_1-Muckenhoupt potentials; De Giorgi-Nash-Moser theory

Full Text: DOI

References:

[1] Aizenman, M.; Simon, B., Brownian motions and Harnack inequality for Schrödinger operators, Commun. Pure Appl. Math., 35, 209-273 (1982) · Zbl 0459.60069

[2] Bourgain, J.; Brezis, H.; Mironescu, P., Limiting embedding theorems for \(W^{s , p}\) when \(s \uparrow 1\) and applications, J. Anal. Math., 87, 77-101 (2002) · Zbl 1029.46030

[3] Chiarenza, F.; Fabes, E.; Garofalo, N., Harnack's inequality for Schrödinger operators and the continuity of solutions, Proc. Am. Math. Soc., 98, 3, 415-425 (1986) · Zbl 0626.35022

[4] Choi, W.; Kim, Y.-C., The Malgrange-Ehrenpreis theorem for nonlocal Schrödinger operators with certain potentials, Commun. Pure Appl. Anal., 17, 1993-2010 (2018) · Zbl 06923931

[5] Choi, W.; Kim, Y.-C., \(L^p\)-mapping properties for nonlocal Schrödinger operators with certain potentials, Discrete Contin. Dyn. Syst., Ser. A, 38, 5811-5834 (2018) · Zbl 1400.45011

[6] Di Castro, A.; Kuusi, T.; Palatucci, G., Nonlocal Harnack inequalities, J. Funct. Anal., 267, 1807-1836 (2014) · Zbl 1302.35082

[7] Di Castro, A.; Kuusi, T.; Palatucci, G., Local behavior of fractional p-minimizers, Ann. Inst. Henri Poincaré, Anal. Non Linéaire, 33, 5, 1279-1299 (2016) · Zbl 1355.35192

[8] Di Nezza, E.; Palatucci, G.; Valdinoci, E., Hitchhiker's guide to the fractional Sobolev spaces, Bull. Sci. Math., 136, 521-573 (2012) · Zbl 1252.46023

[9] Fefferman, C., The uncertainty principle, Bull. Am. Math. Soc. (N.S.), 9, 129-206 (1983) · Zbl 0526.35080

[10] Felsinger, M.; Kassmann, M., Local regularity for parabolic nonlocal operators, Commun. Partial Differ. Equ., 38, 1539-1573 (2013) · Zbl 1277.35090

[11] Garcia-Cuerva, J.; Rubio De Francia, J. L., Weighted Norm Inequalities and Related Topics (1985), North-Holland · Zbl 0578.46046

[12] Giaquinta, M.; Giusti, E., On the regularity of the minima of variational integrals, Acta Math., 148, 31-46 (1982) · Zbl 0494.49031

[13] Giusti, E., Direct Methods in the Calculus of Variations (2003), World Scientific Publishing Co., Inc.: World Scientific Publishing Co., Inc. River Edge · Zbl 1028.49001

[14] Grafakos, L., Classical and Modern Fourier Analysis (2004), Prentice Hall, Pearson Education Inc. · Zbl 1148.42001

[15] Hinz, A. M.; Kalf, H., Subsolution estimates and Harnack's inequality for Schrödinger operators, J. Reine Angew. Math., 404, 118-134 (1990) · Zbl 0779.35026

[16] Y.-C. Kim, Hölder continuity of weak solutions to nonlocal Schrödinger equations with \(A_1\)-Muckenhoupt potentials, submitted for publication.

[17] Kim, Y.-C., Nonlocal Harnack inequalities for nonlocal heat equations, J. Differ. Equ., 267, 6691-6757 (2019) · Zbl 07104401

[18] Kim, Y.-C., Local properties for weak solutions of nonlocal heat equations, Nonlinear Anal. TMA, 192, Article 111689 pp. (2020) · Zbl 1439.35183

[19] Y.-C. Kim, The existence of a fundamental solution for nonlocal Schrödinger operators with \(A_1\)-Muckenhoupt potentials, submitted for publication.

[20] Kinnunen, J.; Shanmugalingam, N., Regularity of quasi-minimizers on metric spaces, Manuscr. Math., 105, 401-423 (2001) · Zbl 1006.49027

[21] Krylov, N. V.; Safonov, M. V., Certain properties of solutions of parabolic equations with measurable coefficients, Izv. Akad. Nauk SSSR, 40, 161-175 (1980), (in Russian)

[22] Laskin, N., Fractional Schrödinger equation, Phys. Rev. E, 66, Article 056108 pp. (2002)

[23] Maz'ya, V.; Shaposhnikova, T., On the Bourgain, Brezis and Mironescu theorem concerning limiting embeddings of fractional Sobolev spaces, J. Funct. Anal., 201, 298-300 (2003)

[24] Murata, M., Structure of positive solutions to \((- \operatorname{\Delta} + V) u = 0\) in \(\mathbb{R}^n\), Duke Math. J., 53, 4, 869-943 (1986) · Zbl 0624.35023

[25] Servadei, R.; Valdinoci, E., Variational methods for non-local operators of elliptic type, Discrete Contin. Dyn. Syst., 33, 2105-2137 (2013) · Zbl 1303.35121

[26] Shen, Z., On the Neumann problem for Schrödinger operators in Lipschitz domains, Indiana Univ. Math. J., 43, 1, 143-176 (1994) · Zbl 0798.35044

[27] Shen, Z., \(L^p\) estimates for Schrödinger operators with certain potentials, Ann. Inst. Fourier, 45, 2, 513-546 (1995) · Zbl 0818.35021

[28] Simon, B., Schrödinger semigroups, Bull. Am. Math. Soc., 7, 3, 447-526 (1982) · Zbl 0524.35002