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Constitutive modelling for the Mullins effect with permanent set and induced anisotropy in particle-filled rubbers. (English) [Zbl 1481.74129](#)

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Summary: Rubber industry plays an important part in various application fields and particle-filled rubbers with reinforced mechanical properties are very essential. The stress softening during cyclic loading, known as the Mullins effect, are evident in filled rubbers. The Mullins softening can induce permanent set and anisotropy. It is crucial to understand these phenomena well. In this paper, we propose a new constitutive model that integrates the micro-sphere model with the network alteration theory. The physical mechanisms of the Mullins effect in particle-filled rubbers are analyzed elaborately and comprehensively to propose new directional damage equations for the network alteration parameters. Both the softening effects in stretched directions and the stiffening effects in contracted directions are taken into account in the modelling. The resulting theoretical model is then applied to predict the stress softening, the residual stretch and the induced anisotropy in various filled rubbers. Moreover, the prediction effect of the current model is compared with those by applying two existing directional macromolecular models. The comparisons with other model predictions and with the experimental data are then analyzed intensively to reveal the underlying mechanisms of the Mullins effect in the different filled rubbers. The results demonstrate the capability of the proposed model to quantitatively predict the Mullins effect with permanent set and induced anisotropy and to clearly reveal the major mechanisms of the Mullins effect for various filled rubbers.

MSC:

74E30 Composite and mixture properties

Keywords:

constitutive model; directional damage evolution; Mullins effect; permanent set; induced anisotropy; particle-filled rubbers

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