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Asymptotics for the concentrated field between closely located hard inclusions in all dimensions. (English) Zbl 1478.78042

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Summary: When hard inclusions are frequently spaced very closely, the electric field, which is the gradient of the solution to the perfect conductivity equation, may be arbitrarily large as the distance between two inclusions goes to zero. In this paper, our objectives are two-fold: first, we extend the asymptotic expansions of *H. Li* et al. [Multiscale Model. Simul. 17, No. 3, 899–925 (2019; Zbl 1426.78022)] to the higher dimensions greater than three by capturing the blow-up factors in all dimensions, which consist of some certain integrals of the solutions to the case when two inclusions are touching; second, our results answer the optimality of the blow-up rate for any $m, n \geq 2$, where m and n are the parameters of convexity and dimension, respectively, which is only partially solved in [*H. Li*, SIAM J. Math. Anal. 52, No. 4, 3350–3375 (2020; Zbl 1447.35140)].

MSC:

78A48 Composite media; random media in optics and electromagnetic theory
35B44 Blow-up in context of PDEs
35B65 Smoothness and regularity of solutions to PDEs
35J25 Boundary value problems for second-order elliptic equations
35C20 Asymptotic expansions of solutions to PDEs

Keywords:

asymptotic expansions; perfect conductivity problem; m-convex inclusions; optimal blow-up rate

Full Text: DOI

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