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**Periodic propagation of complex-valued hyperbolic-cosine-Gaussian solitons and breathers with complicated light field structure in strongly nonlocal nonlinear media.** (English)

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**Summary:** In this study, the propagation characteristics of complex-valued hyperbolic-cosine-Gaussian (CVHCG) beams were studied based on the nonlocal nonlinear Schrödinger equation in strongly nonlocal nonlinear media (SNNM). The CVHCG beams exhibited some unique propagation characteristics. By adjusting the complex-valued parameters, CVHCG beams can propagate with different forms in SNNM, including Gaussian-like, nearly flat-topped, multi-peak, and four-peak forms. CVHCG beams can form shape-invariant solitons and breathers under certain incident parameters. In addition, CVHCG beams can also form generalized shape-variant high-order spatial solitons and breathers. In general, the beam width and light intensity pattern of the CVHCG beams always change periodically. A complete theoretical model was constructed, and the expressions for the propagation, light intensity, and second-order moment beam width were obtained analytically. Some typical propagation characteristics were demonstrated via numerical simulations. The results of this study can be extended to investigate other complex-valued beams.

**MSC:**

- 35Q55 NLS equations (nonlinear Schrödinger equations)
- 35Q60 PDEs in connection with optics and electromagnetic theory
- 78A60 Lasers, masers, optical bistability, nonlinear optics
- 78A40 Waves and radiation in optics and electromagnetic theory
- 35C08 Soliton solutions
- 37K40 Soliton theory, asymptotic behavior of solutions of infinite-dimensional Hamiltonian systems

**Keywords:**

nonlinear Schrödinger equation; high-order solitons; periodic propagation

**Full Text:** [DOI](#)

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