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From the backward Kolmogorov PDE on the Wasserstein space to propagation of chaos for McKean-Vlasov SDEs. (English. French summary) Zbl 1481.60105

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Summary: This article is a continuation of our first work ["Well-posedness for some non-linear diffusion processes and related PDE on the Wasserstein space", J. Math. Pures Appl. (to appear)]. We here establish some new quantitative estimates for propagation of chaos of non-linear stochastic differential equations in the sense of McKean-Vlasov. We obtain explicit error estimates, at the level of the trajectories, at the level of the semi-group and at the level of the densities, for the mean-field approximation by systems of interacting particles under mild regularity assumptions on the coefficients. A first order expansion for the difference between the densities of one particle and its mean-field limit is also established. Our analysis relies on the well-posedness of classical solutions to the backward Kolmogorov partial differential equations defined on the strip $[0,T]\times\mathbb{R}^d\times\mathcal{P}_2(\mathbb{R}^d)$, $\mathcal{P}_2(\mathbb{R}^d)$ being the Wasserstein space, that is, the space of probability measures on $\mathbb{R}^d$ with a finite second-order moment and also on the existence and uniqueness of a fundamental solution for the related parabolic linear operator here stated on $[0,T]\times\mathcal{P}_2(\mathbb{R}^d)$.

MSC:

60H10	Stochastic ordinary differential equations (aspects of stochastic analysis)
93E03	Stochastic systems in control theory (general)
60H30	Applications of stochastic analysis (to PDEs, etc.)
35K40	Second-order parabolic systems

Keywords:

McKean-Vlasov stochastic differential equations; propagation of chaos; backward Kolmogorov partial differential equation; Wasserstein space

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