

Yang, Shuang; Li, Yangrong

Numerical attractors and approximations for stochastic or deterministic sine-Gordon lattice equations. (English) [Zbl 07427465](#)

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Summary: First, we apply the implicit Euler scheme to discretize the sine-Gordon lattice equation (possessing a global attractor) and prove the existence of a numerical attractor for the time-discrete sine-Gordon lattice system with small step sizes. Second, we establish the upper semi-convergence from the numerical attractor towards the global attractor when the step size tends to zero. Third, we establish the upper semi-convergence from the random attractor of the stochastic sine-Gordon lattice equation to the global attractor when the intensity of noise goes to zero. Fourth, we show the finitely dimensional approximations of the three (numerical, random and global) attractors as the dimension of the state space goes to infinity. In a word, we establish four paths of convergence of finitely dimensional (numerical and random) attractors towards the global attractor.

MSC:

[34D45](#) Attractors of solutions to ordinary differential equations

[37L60](#) Lattice dynamics and infinite-dimensional dissipative dynamical systems

[65L20](#) Stability and convergence of numerical methods for ordinary differential equations

Keywords:

sine-Gordon lattice; implicit Euler scheme; numerical attractor; random attractor; finite-dimensional approximation

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