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Bayesian approach to inverse time-harmonic acoustic obstacle scattering with phaseless data generated by point source waves. (English) Zbl 07427314

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Summary: This paper concerns the Bayesian approach to inverse acoustic scattering problems of inferring the position and shape of a sound-soft obstacle from phaseless far-field data generated by two-dimensional point source waves. Given the total number of obstacle parameters, the Markov chain Monte Carlo (MCMC) method is employed to reconstruct the boundary of the obstacle in a high-dimensional space, which usually leads to slow convergence and prohibitively high computational cost. We use the Gibbs sampling and preconditioned Crank-Nicolson (pCN) algorithm with random proposal variance to improve the convergence rate, and design an effective strategy for the surrogate model constructed by the generalized polynomial chaos (gPC) method to reduce the computational cost of MCMC. Numerical examples are provided to illustrate the effectiveness of the proposed method.

MSC:

35R30	Inverse problems for PDEs
35P25	Scattering theory for PDEs
62F15	Bayesian inference
78A46	Inverse problems (including inverse scattering) in optics and electromagnetic theory

Keywords:

inverse scattering; Helmholtz equation; phaseless far-field data; Bayesian inference; MCMC

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