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Existence of three solutions for Kirchhoff-type three-point boundary value problems. (English) [Zbl 07426781](#)

Hacet. J. Math. Stat. 50, No. 2, 304-317 (2021)

Summary: The present paper is an attempt to investigate the multiplicity results of solutions for a three-point boundary value problem of Kirchhoff-type. Indeed, we will use variational methods for smooth functionals, defined on the reflexive Banach spaces in order to achieve the existence of at least three solutions for the equation. Finally, by presenting one example, we will ensure the applicability of our results.

MSC:

[35J20](#) Variational methods for second-order elliptic equations

[34B15](#) Nonlinear boundary value problems for ordinary differential equations

Keywords:

three solutions; three-point boundary value problem; second-order Kirchhoff-type equation; Lipschitz condition; variational methods; critical point theory

Full Text: [DOI](#)

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