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**Optical solitons for Chen-Lee-Liu equation with two spectral collocation approaches.** (English) [Zbl 1478.78053](#)  
*Comput. Math. Math. Phys.* 61, No. 9, 1432-1443 (2021).

**Summary:** This paper revisits the study of optical solitons that is governed by one of the three forms of derivative nonlinear Schrödinger's equation that is also known as Chen-Lee-Liu model. This model is investigated by the aid of fully shifted Jacobi's collocation method with two independent approaches. The first is discretization of the spatial variable, while the other is discretization of the temporal variable. It is concluded that the method of the current paper is far more efficient and reliable for the considered model. Numerical results illustrate the performance efficiency of the algorithm. The results also point out that the scheme can lead to spectral accuracy of the studied model.

**MSC:**

- [78A60](#) Lasers, masers, optical bistability, nonlinear optics
- [35Q55](#) NLS equations (nonlinear Schrödinger equations)
- [35Q41](#) Time-dependent Schrödinger equations and Dirac equations
- [35C08](#) Soliton solutions
- [78M22](#) Spectral, collocation and related methods applied to problems in optics and electromagnetic theory
- [65M70](#) Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs
- [65N35](#) Spectral, collocation and related methods for boundary value problems involving PDEs
- [33C45](#) Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

**Keywords:**

Chen-Lee-Liu equation; collocation method; shifted Jacobi-Gauss-Lobatto quadrature; shifted Jacobi-Gauss-Radau quadrature

**Full Text:** [DOI](#)

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