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The Talbot effect as the fundamental solution to the free Schrödinger equation. (English)

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Summary: The Talbot effect is usually modeled using the Helmholtz equation, but its main experimental features are captured by the solution to the free Schrödinger equation with the Dirac comb as initial datum. This simplified description is a consequence of the paraxial approximation in geometric optics. However, it is a heuristic approximation that is not mathematically well justified, so *K. I. Oskolkov* raised in [Springer Ser. Comput. Math. 19, 353–402 (1992; Zbl 0815.42003)] the problem of “mathematizing” it. We show that it holds exactly in the sense of distributions.

MSC:

35Q41 Time-dependent Schrödinger equations and Dirac equations

46F10 Operations with distributions and generalized functions

35J05 Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation

78A05 Geometric optics

Keywords:

Talbot effect; paraxial approximation; free Schrödinger equation; Dirac comb

Full Text: [DOI](#) [arXiv](#)

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