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Local versus nonlocal constitutive theories of nonequilibrium thermodynamics: the Guyer-Krumhansl equation as an example. (English) Zbl 1479.80003

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On the example of the celebrated Grad's 13-moment system of kinetic theory of rarefied gases and phonon hydrodynamics, it is proved that the constitutive equations of nonequilibrium thermodynamics must be nonlocal. A thermodynamic model of Guyer-Krumhansl heat-transport equation is derived within the frame of weakly nonlocal Rational Thermodynamics. The constitutive equation for the entropy flux is analyzed as well. Some nonlinear generalizations of Maxwell-Cattaneo equation are studied in view of the experiments on thermal wave propagation. The paper arose from a critique of the statement [*T. Ruggeri*, *Q. Appl. Math.* 70, No. 3, 597–611 (2012; [Zbl 1421.74011](#))] that the constitutive equations of continuum thermodynamics cannot be nonlocal, and that the Navier-Stokes-Fourier system can be obtained by the Grad system by a limit procedure. For the exemplary Guyer-Krumhansl system, both the statements have been refuted by proving that (i) the limit process referred there to obtain the Navier-Stokes-Fourier system by the classical Grad system, holds only for a very small subset of all known fluids; (ii) both the Navier-Stokes-Fourier and the Guyer-Krumhansl systems can be obtained by some new closure processes of moments system which put the models beyond the Rational Extended Thermodynamics (RET). The models developed in this paper represent practical examples of the efficacy of nonlocal constitutive equations, which allow to represent both hyperbolic and parabolic situations, without needing of any approximation and regularization procedure.

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MSC:

- [80A17](#) Thermodynamics of continua
- [80A10](#) Classical and relativistic thermodynamics
- [80A19](#) Diffusive and convective heat and mass transfer, heat flow
- [82C35](#) Irreversible thermodynamics, including Onsager-Machlup theory
- [76P05](#) Rarefied gas flows, Boltzmann equation in fluid mechanics
- [78A25](#) Electromagnetic theory (general)
- [35Q79](#) PDEs in connection with classical thermodynamics and heat transfer
- [35Q82](#) PDEs in connection with statistical mechanics
- [35Q20](#) Boltzmann equations
- [35Q60](#) PDEs in connection with optics and electromagnetic theory

Keywords:

Guyer-Krumhansl equation; Grad's 13-moment system; nonlocal constitutive equations; thermal wave speed

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