

Nguyen, Thanh-Hieu; Trong, Dang Duc; Vo, Hoang-Hung

Spreading of two competing species in advective environment governed by free boundaries with a given moving boundary. (English) Zbl 1477.35031

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Summary: In this paper, we study a free boundary problem of two competing species in the left-shifting environment. This model may be used to describe the interaction of the spreading phenomena of two competing species over a one dimensional habitat influenced by an external effect such as the effect of global warming. Here, we assume that only the habitat of inferior competitor is eroded away by the left moving boundary at constant speed c and we consider the how its spreading influences to the spreading of the superior competitor. We prove, as $c_2^* < c < c_1^*$, a trichotomy result: (i) vanishing, (ii) spreading, or (iii) transition for inferior competitor influenced by an advection term caused by the left-shifting boundary and vanishing for superior competitor while both species go extinct in the long run as $c_2^* < c_1^* < c$. This extends the result of *H. Matsuzawa* [Commun. Pure Appl. Anal. 17, No. 5, 1821–1852 (2018; Zbl 1397.35348)] in two aspects: the model is considered for the two competing species and it takes into account the influence of the drift term caused by the the effect of left-shifting boundary.

MSC:

- 35B40 Asymptotic behavior of solutions to PDEs
- 35B50 Maximum principles in context of PDEs
- 35K51 Initial-boundary value problems for second-order parabolic systems
- 35K57 Reaction-diffusion equations
- 35R35 Free boundary problems for PDEs
- 47G20 Integro-differential operators

Keywords:

Fisher-KPP equation; advection; long time behavior; spreading speed; asymptotic profile

Full Text: [DOI](#)

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