

Jornet, Marc

Exact solution to a multidimensional wave equation with delay. (English) Zbl 07425035
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Summary: This paper deals with a mixed problem for the wave equation with discrete delay $\tau > 0$,

$$u_{tt}(t, \mathbf{x}) = a_1^2 \Delta_{\mathbf{x}} u(t, \mathbf{x}) + a_2^2 \Delta_{\mathbf{x}} u(t - \tau, \mathbf{x}) + b_1 u(t, \mathbf{x}) + b_2 u(t - \tau, \mathbf{x}), \quad t > \tau, 0 \leq \mathbf{x} \leq \mathbf{l},$$

with Dirichlet boundary conditions. The exact infinite series solution is constructed by the method of separation of variables, where the time-dependent functions of the decomposition satisfy second-order delay differential equations. Our approach is based on and extends the work by *F. Rodríguez* et al. [ibid. 219, No. 6, 3178–3186 (2012; [Zbl 1309.35045](#))].

MSC:

[35R10](#) Partial functional-differential equations

[35C05](#) Solutions to PDEs in closed form

[35C10](#) Series solutions to PDEs

Keywords:

[delay wave equation](#); [exact series solution](#); [separation of variables](#); [second-order delay differential equation](#)

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