Durdiev, U. D.
Problem of determining the reaction coefficient in a fractional diffusion equation. (English. Russian original) Zbl 1477.35311
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Summary: For a fractional diffusion equation with reaction coefficient depending only on the first two components of the spatial variable $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ and on time $t \geq 0$, we consider the inverse problem of determining this coefficient under the assumption that the initial value at $t = 0$ is known for the solution of the equation and the boundary value at $x_3 = 0$ is given as an additional condition. This inverse problem is reduced to equivalent integral equations, and we apply the contraction mapping principle to prove the existence of solutions of these equations. Local existence and global uniqueness theorems are proved. We also obtain a stability estimate for the solution of the inverse problem.

MSC:

35R30 Inverse problems for PDEs
35K20 Initial-boundary value problems for second-order parabolic equations
35R11 Fractional partial differential equations

Keywords:

equivalent integral equation; contraction mapping principle

Full Text: DOI

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