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Stable knots and links in electromagnetic fields. (English) Zbl 07424943
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Summary: Persistent topological structures in physical systems have become increasingly important over the last years. Electromagnetic fields with knotted field lines play a special role among these, since they can be used to transfer their knottedness to other systems like plasmas and quantum fluids. In null electromagnetic fields the electric and the magnetic field lines evolve like unbreakable elastic filaments in a fluid flow. In particular, their topology is preserved for all time, so that all knotted closed field lines maintain their knot type. We use an approach due to Bateman to prove that for every link L there is such an electromagnetic field that satisfies Maxwell's equations in free space and that has closed electric and magnetic field lines in the shape of L for all time. The knotted and linked field lines turn out to be projections of real analytic Legendrian links with respect to the standard contact structure on the 3-sphere.

MSC:

- 35Qxx** Partial differential equations of mathematical physics and other areas of application
- 78Axx** General topics in optics and electromagnetic theory
- 57K10** Knot theory
- 57R17** Symplectic and contact topology in high or arbitrary dimension

Keywords:

[Legendrian knots](#)

Full Text: [DOI](#) [arXiv](#)

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