

Dostalík, Mark; Matyska, Ctirad; Průša, Vít

Weakly nonlinear analysis of Rayleigh-Bénard convection problem in extended Boussinesq approximation. (English) [Zbl 07424237](#)

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Summary: We investigate Rayleigh-Bénard convection problem in an extended Boussinesq approximation suitable for conditions in the Earth's mantle. The aim is to evaluate the influence of depth-dependent material parameters, dissipation, adiabatic heating/cooling and heat sources on qualitative characteristics of thermal convection. We identify the critical values of dimensionless parameters that determine the onset of convection, and we characterize the dominating convection patterns in marginally supercritical states. These issues are addressed by the application of linear stability analysis and weakly nonlinear analysis. We have found that the character of convection differs substantially from the standard case of Rayleigh-Bénard convection.

MSC:

76E06 Convection in hydrodynamic stability

76E20 Stability and instability of geophysical and astrophysical flows

80A20 Heat and mass transfer, heat flow (MSC2010)

Keywords:

[thermal convection](#); [extended Boussinesq approximation](#); [weakly nonlinear analysis](#)

Software:

[Matlab](#); [Differentiation Matrix Suite](#)

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References:

- [1] Chandrasekhar, S., Hydrodynamic and hydromagnetic stability, The International Series of Monographs on Physics (1961), Clarendon Press, Oxford · [Zbl 0142.44103](#)
- [2] Matyska, C.; Yuen, D. A., Lower mantle material properties and convection models of multiscale plumes, Geological Society of America Special Papers, 137-163 (2007)
- [3] Rajagopal, K.; Saccomandi, G.; Vergori, L., Stability analysis of the rayleigh-bénard convection for a fluid with temperature- and pressure-dependent viscosity, *Z. angew. Math. Phys.*, 60, 4, 739-755 (2009) · [Zbl 1169.76023](#)
- [4] Rajagopal, K. R.; Saccomandi, G.; Vergori, L., On the oberbeck-boussinesq approximation for fluids with pressure dependent viscosities, *Nonlinear Anal. Real World Appl.*, 10, 2, 1139-1150 (2009) · [Zbl 1167.76368](#)
- [5] Rosenblat, S., Thermal convection in a viscoelastic liquid, *J. Non-Newtonian Fluid Mech.*, 21, 2, 201-223 (1986) · [Zbl 0624.76007](#)
- [6] Demir, H., Rayleigh-benard convection of viscoelastic fluid, *Appl. Math. Comput.*, 136, 2, 251-267 (2003) · [Zbl 1135.76325](#)
- [7] Demir, H.; Akyildiz, F., Unsteady thermal convection of a non-newtonian fluid, *Int. J. Eng. Sci.*, 38, 17, 1923-1938 (2000) · [Zbl 1210.76070](#)
- [8] Bouteraa, M.; Nouar, C.; Plaut, E.; Metivier, C.; Kalck, A., Weakly nonlinear analysis of rayleigh-bénard convection in shear-thinning fluids: nature of the bifurcation and pattern selection, *J. Fluid Mech.*, 767, 696734 (2015) · [Zbl 1335.76047](#)
- [9] Bouteraa, M.; Nouar, C., Rayleigh-bénard convection in non-newtonian carreau fluids with arbitrary conducting boundaries, *Int. Commun. Heat Mass*, 76, 77-84 (2016)
- [10] Bouteraa, M.; Nouar, C., Weakly nonlinear analysis of rayleigh-bénard convection in a non-newtonian fluid between plates of finite conductivity: influence of shear-thinning effects, *Phys. Rev. E*, 92, 063017 (2015)
- [11] Gupta, G.; Massoudi, M., Flow of a generalized second grade fluid between heated plates, *Acta Mech.*, 99, 1-4, 21-33 (1993) · [Zbl 0774.76005](#)
- [12] Straughan, B., Energy stability in the Bénard problem for a fluid of second grade, *Z. angew. Math. Phys.*, 34, 4, 502-509 (1983) · [Zbl 0557.76011](#)
- [13] Franchi, F.; Straughan, B., Convection, stability and uniqueness for a fluid of third grade, *Int. J. Non-Linear Mech.*, 23, 5, 377-384 (1988) · [Zbl 0673.76008](#)

- [14] Massoudi, M.; Vaidya, A.; Wulandana, R., Natural convection flow of a generalized second grade fluid between two vertical walls, *Nonlinear Anal. Real World Appl.*, 9, 1, 80-93 (2008) · [Zbl 1130.76077](#)
- [15] Aghighi, M. S.; Ammar, A.; Metivier, C.; Gharagozlu, M., Rayleigh-bénard convection of casson fluids, *Int. J. Therm. Sci.*, 127, 79-90 (2018)
- [16] Cross, M. C., Derivation of the amplitude equation at the rayleigh-bénard instability, *Phys. Fluids*, 23, 9, 1727-1731 (1980) · [Zbl 0444.76066](#)
- [17] Fujimura, K., Methods of centre manifold and multiple scales in the theory of weakly nonlinear stability for fluid motions, *Proc. R. Soc. A: Math. Phys. Eng. Sci.*, 434, 1892, 719-733 (1991) · [Zbl 0746.76047](#)
- [18] Fujimura, K., Centre manifold reduction and the stuart-landau equation for fluid motions, *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.*, 453, 181-203 (1997) · [Zbl 0871.76024](#)
- [19] Kagei, Y.; Růžička, M.; Thäter, G., Natural convection with dissipative heating, *Commun. Math. Phys.*, 214, 287-313 (2000) · [Zbl 0986.76083](#)
- [20] Rajagopal, K. R.; Růžička, M.; Srinivasa, A. R., On the oberbeck-boussinesq approximation, *Math. Models Methods Appl. Sci.*, 6, 8, 1157-1167 (1996) · [Zbl 0883.76078](#)
- [21] Zeytounian, R., Joseph boussinesq and his approximation: a contemporary view, *C. R. Mec.*, 331, 8, 575-586 (2003) · [Zbl 1222.76005](#)
- [22] Schubert, G.; Turcotte, D. L.; Olsen, P., *Mantle convection in the earth and planets*, Cambridge Monographs on Mechanics (2001), Cambridge University Press
- [23] Barus, C., Isotherms, isopiestic and isometrics relative to viscosity, *Amer. J. Sci.*, 45, 87-96 (1893)
- [24] Bair, S.; Kottke, P., Pressure-viscosity relationships for elastohydrodynamics, *Tribol. Trans.*, 46, 3, 289-295 (2003)
- [25] Trefethen, L. N., *Spectral methods in MATLAB, Software, Environments, and Tools*, 10 (2000), Society for Industrial and Applied Mathematics (SIAM): Society for Industrial and Applied Mathematics (SIAM) Philadelphia, PA · [Zbl 0953.68643](#)
- [26] Boyd, J. P., *Chebyshev and fourier spectral methods* (2001), Dover Publications Inc.: Dover Publications Inc. Mineola, NY · [Zbl 0994.65128](#)
- [27] Weideman, J. A.; Reddy, S. C., A MATLAB differentiation matrix suite, *ACM Trans. Math. Softw.*, 26, 4, 465-519 (2000)
- [28] Di Prima, R. C.; Habetler, G. J., A completeness theorem for non-selfadjoint eigenvalue problems in hydrodynamic stability, *Arch. Ration. Mech. Anal.*, 34, 3, 218-227 (1969) · [Zbl 0181.54703](#)
- [29] Bhadauria, B.; Siddheshwar, P.; Kumar, J.; Suthar, O., Weakly nonlinear stability analysis of temperature/gravity-modulated stationary rayleigh-bénard convection in a rotating porous medium, *Transp. Porous. Med.*, 92, 633647 (2011)

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