

**Ghisi, Marina; Giraudo, Chiara; Gobbino, Massimo; Haraux, Alain**  
**Sharp ultimate velocity bounds for the general solution of some linear second order evolution equation with damping and bounded forcing.** (English) Zbl 07423283  
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In the paper under review, the authors prove the equivalence of different notions of optimal bound of solutions to a class of abstract evolution equations (Theorem 2.2). Furthermore, they investigate the optimal velocity bound for both scalar equations

$$u''(t) + cu'(t) + bu(t) = f(t)$$

and vector equations

$$u''(t) + cu'(t) + Au(t) = f(t).$$

The paper is well-written and the subject is of a significant interest.

Reviewer: Nicolae Lupa (Timișoara)

#### MSC:

- [34G10](#) Linear differential equations in abstract spaces
- [37C60](#) Nonautonomous smooth dynamical systems
- [34D05](#) Asymptotic properties of solutions to ordinary differential equations
- [34C11](#) Growth and boundedness of solutions to ordinary differential equations

#### Keywords:

second-order differential equation; dissipative equation; forcing term; asymptotic behavior of solutions; bounded solutions; ultimate bound

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