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An effective method for calculating elasto-plastic contact pressure and contact patch size under elliptical, circular and line contact conditions. (English) Zbl 1481.74581
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Summary: According to the Hertz's contact theory (HCT) and considering the strain hardening of materials, an effective calculation method is proposed for solving the elasto-plastic contact pressure and the contact patch size under arbitrary smooth and continuous contact conditions. Firstly, it is assumed that the outer and inner edges of the contact patch are elastic and elasto-plastic contact zones, respectively, the contact pressure in the elastic contact zone satisfies the HCT, while the contact pressure in the elasto-plastic contact zone is the superposition of a constant contact pressure and several small ellipsoidal distribution contact pressures. Then, the explicit expressions of contact patch size and contact pressure under the elasto-plastic elliptical, circular and line contact conditions are derived, respectively. Subsequently, the accuracy and applicability of the proposed method are evaluated by comparing the predicted results with the finite element simulations. Finally, the accuracy of the proposed method is further verified by comparing the predicted results with the typical results in literature. It is shown that the proposed method has high calculation accuracy, and the maximum relative errors are 9.32% for the elliptical contact, 4.61% for the circular contact and 11.84% for the line contact, respectively. Meanwhile, the proposed method can effectively predict the contact pressure distribution under the elasto-plastic elliptical, circular and line contacts, and shows a general applicability for different contact materials, contact-body's sizes and normal external loads, as well as for the elasto-plastic wheel-rail contact analysis.

MSC:

[74M15](#) Contact in solid mechanics

[74C05](#) Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)

Keywords:

[elasto-plastic contact](#); [contact pressure](#); [contact patch size](#); [Hertz's contact theory](#); [prediction accuracy](#)

Software:

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References:

- [1] Li, S., A mathematical model and numeric method for contact analysis of rolling bearings, *Mech. Mach. Theory*, 119, 61-73 (2018)
- [2] Li, F. K.; Hu, W. P.; Meng, Q. C.; Zhan, Z. X.; Shen, F., A new damage-mechanics-based model for rolling contact fatigue analysis of cylindrical roller bearing, *Tribol. Int.*, 120, 105-114 (2018)
- [3] Piotrowski, J.; Kik, W., A simplified model of wheel/rail contact mechanics for non-Hertzian problems and its application in rail vehicle dynamic simulations, *Veh. Syst. Dyn.*, 46, 27-48 (2008)
- [4] Masoudi Nejad, R.; Shariati, M.; Farhangdoost, K., Effect of wear on rolling contact fatigue crack growth in rails, *Tribol. Int.*, 94, 118-125 (2016)
- [5] Daves, W.; Kubin, W.; Scheriau, S.; Pletz, M., A finite element model to simulate the physical mechanisms of wear and crack initiation in wheel/rail contact, *Wear*, 366-367, 78-83 (2016)
- [6] Hou, X. Y.; Fang, Z. D.; Zhang, X. J., Static contact analysis of spiral bevel gear based on modified VFIFE (vector form intrinsic finite element) method, *Appl. Math. Model.*, 60, 192-207 (2018) · [Zbl 1480.70003](#)
- [7] Zhang, B. Y.; Liu, H. J.; Bai, H. Y.; Zhu, C. C.; Wu, W., Ratchetting-multi-axial fatigue damage analysis in gear rolling contact considering tooth surface roughness, *Wear*, 428-429, 137-146 (2019)
- [8] Johnson, K. L., *Contact Mechanics* (1985), Cambridge University Press: Cambridge University Press Cambridge, (book) · [Zbl 0599.73108](#)
- [9] Kalker, J. J., *Three-Dimensional Elastic Bodies in Rolling Contact* (1990), Kluwer Academic Publishers: Kluwer Academic

Publishers Dordrecht (Netherlands), (Ph. D thesis) · [Zbl 0709.73068](#)

- [10] Jin, X. S.; Liu, Q. Y., *Tribology of Wheel and Rail* (2004), China Railway Press: China Railway Press Beijing, (in Chinese)
- [11] Carter, F. W., On the action of a locomotive driving wheel, *Proc. R. Soc. Lond.*, 112, 151-157 (1926) · [Zbl 52.0832.05](#)
- [12] Johnson, K. L., The effect of a tangential contact force on the rolling motion of an elastic sphere on a plane, *J. Appl. Mech.*, 25, 339-346 (1958) · [Zbl 0105.18302](#)
- [13] Vermeulen, P. J.; Johnson, K. L., Contact of nonspherical elastic bodies transmitting tangential forces, *J. Appl. Mech.*, 17, 338-340 (1964) · [Zbl 0133.18105](#)
- [14] Shen, Z. Y.; Hedrick, J. K.; Elkins, J. A., A comparison of alternative creep force models for rail vehicle dynamic analysis, *Veh. Syst. Dyn.*, 12, 79-83 (1983)
- [15] Kalker, J. J., On the rolling contact of two elastic bodies in the presence of dry friction (1967), Delft University: Delft University Delft, (Ph. D thesis)
- [16] Kalker, J. J., A fast algorithm for the simplified theory of rolling contact, *Veh. Syst. Dyn.*, 11, 1-13 (1982)
- [17] Kalker, J. J., *Three-Dimensional Elastic Bodies in Rolling Contact* (1990), Kluwer Academic Press, (book) · [Zbl 0709.73068](#)
- [18] Yaylaçı, M.; Avcar, M., Finite element modeling of contact between an elastic layer and two elastic quarter planes, *Comput. Concr.*, 26, 107-114 (2020)
- [19] Adiyaman, G.; Birinci, A.; Öner, E.; Yaylaçı, M., A receding contact problem between a functionally graded layer and two homogeneous quarter planes, *Acta Mech.*, 227, 1753-1766 (2016) · [Zbl 1341.74125](#)
- [20] Yaylaçı, M.; Terzi, C.; Avcar, M., Numerical analysis of the receding contact problem of two bonded layers resting on an elastic half plane, *Struct. Eng. Mech.*, 72, 775-783 (2019)
- [21] Öner, E.; Yaylaçı, M.; Birinci, A., Analytical solution of a contact problem and comparison with the results from FEM, *Struct. Eng. Mech.*, 54, 607-622 (2015)
- [22] Yaylaçı Eren, U.; Yaylaçı, M.; Ölmez, H.; Birinci, A., Artificial neural network calculations for a receding contact problem, *Comput. Concr.*, 25, 551-563 (2020)
- [23] Wu, C. Y.; Li, L. Y.; Thornton, C., Rebound behaviour of spheres for plastic impacts, *Int. J. Impact Eng.*, 28, 929-946 (2003)
- [24] Yu, W.; Blanchard, J. P., An elastic-plastic indentation model and its solutions, *J. Mater. Res.*, 11, 2358-2367 (2011)
- [25] Zhao, J. Z.; Kan, Q. H.; Fu, P. L.; Kang, G. Z.; Wang, P., An elasto-plastic contact solving method for two spheres, *Acta Mech. Solida Sin.*, 33, 612-634 (2020)
- [26] Yan, W. Y.; Fischer, F. D., Applicability of the Hertz contact theory to rail-wheel contact problems, *Arch. Appl. Mech.*, 70, 255-268 (2000) · [Zbl 0948.74523](#)
- [27] Cao, S. H.; Li, X.; Zhang, S. F.; Wen, L. H.; Jiang, X. Y., Research of the differences between hertz theory and finite element method to analyze the fatigue of wheel/rail contact, *J. Mech. Eng.*, 51, 126-134 (2015), (in Chinese)
- [28] Zhang, H. W.; Zhong, W. X.; Wu, C. H.; Liao, A. H., Some advances and applications in quadratic programming method for numerical modeling of elastoplastic contact problems, *Int. J. Mech. Sci.*, 48, 176-189 (2006) · [Zbl 1099.74072](#)
- [29] Chung, J. C., Elastic-plastic contact analysis of an ellipsoid and a rigid flat, *Tribol. Int.*, 43, 491-502 (2010)
- [30] Dong, X.; Yin, X.; Deng, Q.; Yu, B.; Wang, H.; Weng, P.; Chen, C.; Yuan, H., Local contact behavior between elastic and elastic-plastic bodies, *Int. J. Solids Struct.*, 150, 22-39 (2018)
- [31] Hardy, C.; Baronet, C. N.; Tordion, G. V., The elasto-plastic indentation of a half-space by a rigid sphere, *Int. J. Numer. Methods Eng.*, 3, 451-462 (1971)
- [32] Thornton, C.; Ning, Z., A theoretical model for the stick/bounce behaviour of adhesive, elastic-plastic spheres, *Powder Technol.*, 99, 154-162 (1998)
- [33] Thornton, C.; Cummins, S. J.; Cleary, P. W., On elastic-plastic normal contact force models, with and without adhesion, *Powder Technol.*, 315, 339-346 (2017)
- [34] Thornton, C., Coefficient of restitution for collinear collisions of elastic-perfectly plastic spheres, *J. Appl. Mech.*, 64, 383-386 (1997) · [Zbl 0883.73072](#)
- [35] Brake, M. R. W., An analytical elastic plastic contact model with strain hardening and frictional effects for normal and oblique impacts, *Int. J. Solids Struct.*, 62, 104-123 (2015)
- [36] Zhu, H. B.; Zhao, Y. T.; He, Z. F.; Zhang, R. N.; Ma, S. P., An elastic-plastic contact model for line contact structures, *Sci. China Phys. Mech. Astron.*, 61 (2018)
- [37] Zhu, H. B.; He, Z. F.; Zhao, Y. T.; Ma, S. P., Experimental verification of yield strength of polymeric line contact structures, *Polym. Test.*, 63, 118-125 (2017)
- [38] Zhu, H. B.; He, Z. F.; Jiang, H.; Ma, S. P., Experimental investigation into the failure mechanism of ductile line contact structures, *Mech. Mater.*, 129, 375-380 (2019)
- [39] Yan, W. Y.; Bus so, E. P.; O'Dowd, N. P., A micromechanics investigation of sliding wear in coated components, *Proc. R. Soc. Lond., Ser. A Math. Phys. Eng. Sci.*, 456, 2387-2407 (2000)
- [40] Su, H. Y.; Lv, D.; Li, Y.; Chen, H. K., The discussion on the formula of strain hardening exponent (n-values), *Percutaneous Transl. Coron. Angioplasty PTCA Part A Phys. Test*, 12, 621-623 (2006), +620. (in Chinese)
- [41] Song, Y. Q.; Guan, Z. P.; Ma, P. K.; Song, J. W., Theoretical and experimental standardization of strain hardening index in tensile deformation, *Acta Metall. Sin.*, 42, 673-680 (2006), (in Chinese)

- [42] Wang, Z. J.; Wang, W. Z.; Hu, Y. Z.; Wang, H., A numerical elastic-plastic contact model for rough surfaces, *Tribol. Trans.*, 53, 224-238 (2010)
- [43] Srivastava, J. P.; Sarkar, P. K.; Meesala, V. R.K.; Ranjan, V., Rolling contact fatigue life of rail for different slip conditions, *Latin Am. J. Solids Struct.*, 14, 2243-2264 (2017)
- [44] Cai, W.; Wen, Z. F.; Jin, X. S., Effects of rail gap on wheel-rail contact stresses, *Eng. Mech.*, 9, 173-178 (2006), (in Chinese)

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