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Existence of positive solutions for periodic boundary value problems of second-order impulsive differential equation with derivative in the nonlinearity. (English) [Zbl 07419637](#)

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The paper explores the existence of positive solutions of a periodic boundary value problem of a second-order impulsive differential equation

$$\begin{cases} -u'' + \rho^2 u = f(t, u, u'), t \in J', \\ -\Delta u'|_{t=t_k} = I_k(u(t_k)), k = 1, 2, \dots, m, \\ u(0) = u(2\pi), u'(0) = u'(2\pi), \end{cases}$$

where $J = [0, 2\pi]$, $J' = J \setminus \{t_1, t_2, \dots, t_m\}$. The paper studied the Green's function and its inequalities and properties for the above boundary value problem and established the equivalence of solutions to the BVP and the fixed points of an integral operator. Under different superlinear or sublinear conditions of f , the authors investigated the fixed point index of the operator on defined cones and established existence of positive solutions of the boundary value problem. Examples are provided to illustrate the applicability of the main results.

Reviewer: [Xueyan Liu \(New Orleans\)](#)

MSC:

- [34B15](#) Nonlinear boundary value problems for ordinary differential equations
- [34B18](#) Positive solutions to nonlinear boundary value problems for ordinary differential equations
- [34B37](#) Boundary value problems with impulses for ordinary differential equations
- [47N20](#) Applications of operator theory to differential and integral equations

Keywords:

positive solution; impulsive equation; fixed point index; periodic boundary value problems; second-order differential equation

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