

Liu, Songshu; Feng, Lixin; Zhang, Guilai

An inverse source problem of space-fractional diffusion equation. (English) Zbl 1476.35333
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Summary: This paper is devoted to an inverse space-dependent source problem for space-fractional diffusion equation. Furthermore, we show that this problem is ill-posed in the sense of Hadamard, i.e., the solution (if it exists) does not depend continuously on the data. In addition, we propose a simplified generalized Tikhonov regularization method and prove the corresponding convergence estimates by using a priori regularization parameter choice rule and a posteriori parameter choice rule, respectively. Finally, numerical examples are carried to support the theoretical results and illustrate the effectiveness of the proposed method.

MSC:

35R30 Inverse problems for PDEs

35K15 Initial value problems for second-order parabolic equations

35R11 Fractional partial differential equations

Keywords:

space-fractional diffusion equation; simplified generalized Tikhonov regularization method; a priori parameter choice; a posteriori parameter choice

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