

**Berberich, Jonas P.; Chandrashekar, Praveen; Klingenberg, Christian; Röpke, Friedrich K.** **Second order finite volume scheme for Euler equations with gravity which is well-balanced for general equations of state and grid systems.** (English) Zbl 07418030  
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**Summary:** We develop a second order well-balanced finite volume scheme for compressible Euler equations with a gravitational source term. The well-balanced property holds for arbitrary hydrostatic solutions of the corresponding Euler equations without any restriction on the equation of state. The hydrostatic solution must be known a priori either as an analytical formula or as a discrete solution at the grid points. The scheme can be applied to curvilinear meshes and in combination with any consistent numerical flux function and time stepping routines. These properties are demonstrated on a range of numerical tests.

**MSC:**

**76M12** Finite volume methods applied to problems in fluid mechanics  
**76N15** Gas dynamics (general theory)

Cited in **2** Documents

**Keywords:**

finite volume method; compressible Euler equations; well-balancing; gravity effect; curvilinear mesh; numerical flux function

**Software:**

HE-E1GODF; Chebfun

**Full Text:** [DOI](#) [arXiv](#)

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