Chen, Wenjing; **Rădulescu, Vicenţiu D.**; **Zhang, Binlin**

Fractional Choquard-Kirchhoff problems with critical nonlinearity and Hardy potential.

(English) Zbl 1479.35419
Anal. Math. Phys. 11, No. 3, Paper No. 132, 25 p. (2021).

The authors study a fractional p-Kirchhoff-type problem with fractional p-Laplacian. The existence of a positive weak solution is proved by the use of the concentration-compactness principle and mountain pass theorem.

Reviewer: Leszek Gasiński (Kraków)

MSC:

35J62 Quasilinear elliptic equations
35R11 Fractional partial differential equations
35A01 Existence problems for PDEs: global existence, local existence, non-existence
35J20 Variational methods for second-order elliptic equations

Keywords:

fractional p-Kirchhoff-type equation; Choquard nonlinearity; existence; concentration-compactness principle

Full Text: DOI

References:

[1] Fröhlich, H., Theory of electrical breakdown in ionic crystal, Proc. Roy. Soc. Edinburgh Sect. A, 160, 230-241 (1937)

[2] Pekar, S., Untersuchung uber die Elektronentheorie der Kristalle (1954), Berlin: Akademie Verlag, Berlin · Zbl 0058.45503

[3] Elgart, A.; Schlein, B., Mean field dynamics of boson stars, Commun. Pure Appl. Math., 60, 500-545 (2007) · Zbl 1113.81032 · doi:10.1002/cpa.20134

[4] Giulini, D.; Großardt, A., The Schrödinger-Newton equation as a non-relativistic limit of self-gravitating Klein-Gordon and Dirac fields, Classical Quant. Grav., 29, 215010 (2012) · Zbl 1266.83009 · doi:10.1088/0264-9381/29/21/215010

[5] Jones, KRW, Gravitational self-energy as the litmus of reality, Mod. Phys. Lett. A, 10, 8, 657-668 (1995)· doi:10.1142/S0217732395000703

[6] Schunck, FE; Mielke, EW, General relativistic boson stars, Class. Quantum Grav., 20, R301-R356 (2003) · Zbl 1050.83002 · doi:10.1088/0264-9381/20/20/201

[7] Penrose, R., Quantum computation, entanglement and state reduction, R. Soc. Lond. Philos. Trans. Ser. A Math. Phys. Eng. Sci., 356, 1927-1939 (1998) · Zbl 1152.81659 · doi:10.1098/rsta.1998.0256

[8] Penrose, R.: The road to reality. Alfred A. Knopf Inc., New York, A complete guide to the laws of the Universe (2005) · Zbl 1188.00007

[9] Lieb, E.: Existence and uniqueness of the minimizing solution of Choquard's nonlinear equation. Stud. Appl. Math. 57, 93-105 (1976/1977) · Zbl 0369.35022

[10] Ma, L.; Zhao, L., Classification of positive solitary solutions of the nonlinear Choquard equation, Arch. Ration. Mech. Anal., 195, 455-467 (2010) · Zbl 1185.35260 · doi:10.1007/s00205-008-0208-3

[11] Moroz, V.; Van Schaftingen, J., A guide to the Choquard equation, J. Fixed Point Theor. Appl., 19, 773-813 (2017) · Zbl 1360.35252 · doi:10.1007/s11784-016-0373-1

[12] Cassani, D.; Zhang, J., Choquard-type equations with Hardy-Littlewood-Sobolev upper-critical growth, Adv. Nonlinear Anal., 8, 1184-1212 (2019) · Zbl 1418.35168 · doi:10.1515/anona-2018-0019

[13] Mingqi, X.; Rădulescu, V.; Zhang, B., A critical fractional Choquard-Kirchhoff problem with magnetic field, Commun. Contemp. Math., 21, 1850004 (2019) · Zbl 1416.49012 · doi:10.1142/S0219199718500049

[14] Seok, J., Limit profiles and uniqueness of ground states to the nonlinear Choquard equations, Adv. Nonlinear Anal., 8, 1083-1098 (2019) · Zbl 1419.35066 · doi:10.1515/anona-2017-0182

[15] Marano, S.; Mosconi, S., Asymptotic for optimizers of the fractional Hardy-Sobolev inequality, Commun. Contemp. Math., 21, 1850028 (2019) · Zbl 1430.35006 · doi:10.1142/S0219199718500281

[16] Fiscella, A.; Valdinoci, E., A critical Kirchhoff type problem involving a nonlocal operator, Nonlinear Anal., 94, 156-170 (2014)

· Zbl 1283.35156 · doi:10.1016/j.na.2013.08.011

[17] Autuori, G.; Fiscella, A.; Pucci, P., Stationary Kirchhoff problems involving a fractional elliptic operator and a critical nonlinearity, Nonlinear Anal., 125, 699-714 (2015) · Zbl 1323.35015 · doi:10.1016/j.na.2015.06.014

[18] Molica Bisci, G., Rădulescu, V., Servadei, R.: Variational methods for nonlocal fractional equations. Encyclop. Math. Appl. 162, Cambridge University Press, Cambridge, (2016) · Zbl 1356.49003

[19] Pucci, P.; Xiang, M.; Zhang, B., Multiple solutions for nonhomogeneous Schrödinger-Kirchhoff type equations involving the fractional \(p\)-Laplacian in \({{\mathbb{R}}}^N\), Calc. Var. Partial Differ. Equ., 54, 2785-2806 (2015) · Zbl 1329.35338 · doi:10.1007/s00526-015-0883-5

[20] Pucci, P.; Xiang, M.; Zhang, B., Existence and multiplicity of entire solutions for fractional \(p\)-Kirchhoff equations, Adv. Nonlinear Anal., 5, 27-55 (2016) · Zbl 1334.35395

[21] Wang, L.; Cheng, K.; Zhang, B., A uniqueness result for strong singular Kirchhoff-type fractional Laplacian problems, Appl. Math. Opt., 83, 1859-1875 (2021) · Zbl 1469.35002 · doi:10.1007/s00245-019-09612-y

[22] Xiang, M.; Zhang, B.; Ferrara, M., Existence of solutions for Kirchhoff type problem involving the non-local fractional \(p\)-Laplacian, J. Math. Anal. Appl., 424, 1021-1041 (2015) · Zbl 1317.35286 · doi:10.1016/j.jmaa.2014.11.055

[23] Xiang, M.; Zhang, B.; Rădulescu, V., Multiplicity of solutions for a class of quasilinear Kirchhoff system involving the fractional \(p\)-Laplacian, Nonlinearity, 29, 3186-3205 (2016) · Zbl 1349.35413 · doi:10.1088/0951-7715/29/10/3186

[24] Xiang, M.; Zhang, B.; Zhang, X., A nonhomogeneous fractional \(p\)-Kirchhoff type problem involving critical exponent in \({\mathbb{R}}^N\), Adv. Nonlinear Stud., 17, 611-640 (2017) · Zbl 1372.35348 · doi:10.1515/ans-2016-6002

[25] Fiscella, A.; Pucci, P., Kirchhoff-Hardy fractional problems with lack of compactness, Adv. Nonlinear Stud., 17, 429-456 (2017) · Zbl 1375.35180 · doi:10.1515/ans-2017-6021

[26] Fiscella, A.; Pucci, P., \(p\)-fractional Kirchhoff equations involving critical nonlinearities, Nonlinear Anal. Real World Appl., 35, 350-378 (2017) · Zbl 1372.35335 · doi:10.1016/j.nonrwa.2016.11.004

[27] Fiscella, A.; Pucci, P.; Zhang, B., \(p\)-fractional Hardy-Schrödinger-Kirchhoff systems with critical nonlinearities, Adv. Nonlinear Anal., 8, 1111-1131 (2019) · Zbl 1414.35258 · doi:10.1515/anona-2018-0033

[28] Song, Y.; Shi, S., Existence of infinitely many solutions for degenerate \(p\)-fractional Kirchhoff equations with critical Sobolev-Hardy nonlinearities, Z. Angew. Math. Phys., 68, 68 (2017) · Zbl 1383.35090 · doi:10.1007/s00033-017-0867-8

[29] Song, Y.; Shi, S., On a degenerate \(p\)-fractional Kirchhoff equations with critical Sobolev-Hardy nonlinearities, Mediterr. J. Math., 15, 17 (2018) · Zbl 1390.35090 · doi:10.1007/s00009-017-1062-z

[30] Shen, Z.; Gao, F.; Yang, M., Ground states for nonlinear fractional Choquard equations with general nonlinearities, Math. Methods Appl. Sci., 39, 4082-4098 (2016) · Zbl 1344.35168 · doi:10.1002/mma.3849

[31] Wang, Y., Yang, Y.: Nonlocal problems with critical Choquard nonlinearities, Preprint

[32] Wu, D., Existence and stability of standing waves for nonlinear fractional Schrödinger equations with Hartree type nonlinearity, J. Math. Anal. Appl., 411, 530-542 (2014) · Zbl 1332.35343 · doi:10.1016/j.jmaa.2013.09.054

[33] Pucci, P.; Xiang, M.; Zhang, B., Existence results for Schrödinger-Choquard-Kirchhoff equations involving the fractional \(p\)-Laplacian, Adv. Calc. Var., 12, 253-275 (2019) · Zbl 1431.35233 · doi:10.1515/acv-2016-0049

[34] Chen, W.; Mosconi, S.; Squassina, M., Nonlocal problems with critical Hardy nonlinearity, J. Funct. Anal., 275, 3065-3114 (2018) · Zbl 1402.35113 · doi:10.1016/j.jfa.2018.02.020

[35] Chen, W., Critical fractional \(p\)-Kirchhoff type problem with a generalized Choquard nonlinearity, J. Math. Phys., 59, 121502 (2018) · Zbl 1409.35077 · doi:10.1063/1.5052669

[36] Xiang, M.; Zhang, B.; Rădulescu, V., Superlinear Schrödinger-Kirchhoff type problems involving the fractional \(p\)-Laplacian and critical exponent, Adv. Nonlinear Anal., 9, 690-709 (2020) · Zbl 1427.35340 · doi:10.1515/anona-2020-0021

[37] Ghoussoub, N.; Yuan, C., Multiple solutions for quasi-linear PDEs involving the critical Sobolev and Hardy exponents, Trans. Am. Math. Soc., 352, 5703-5743 (2000) · Zbl 0956.35056 · doi:10.1090/S0002-9947-00-02560-5

[38] Ambrosetti, A.; Rabinowitz, P., Dual variational methods in critical point theory and applications, J. Funct. Anal., 14, 349-381 (1973) · Zbl 0273.49063 · doi:10.1016/0022-1236(73)90051-7

[39] Brasco, L.; Mosconi, S.; Squassina, M., Optimal decay of extremal functions for the fractional Sobolev inequality, Calc. Var. Partial Differ. Equ., 55, 1-32 (2016) · Zbl 1350.46024 · doi:10.1007/s00526-016-0958-y

[40] Lieb, E.; Loss, M., Analysis, Graduate Studies in Mathematics (2001), Providence, Rhode Island: Amer. Math. Soc, Providence, Rhode Island

[41] Mosconi, S.; Squassina, M., Nonlocal problems at nearly critical growth, Nonlinear Anal., 136, 84-101 (2016) · Zbl 1337.35053 · doi:10.1016/j.na.2016.02.012

[42] D'Avenia, P.; Squassina, M., On fractional Choquard equations, Math. Models Methods Appl. Sci., 25, 1447-1476 (2015) · Zbl 1323.35205 · doi:10.1142/S0218202515500384