Dier, Dominik; **Kassmann, Moritz**; **Zacher, Rico**

Discrete versions of the Li-Yau gradient estimate. (English) Zbl 1476.35289

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Summary: We study positive solutions to the heat equation on graphs. We prove variants of the Li-Yau gradient estimate and the differential Harnack inequality. For some graphs, we can show the estimates to be sharp. We establish new computation rules for differential operators on discrete spaces and introduce a relaxation function that governs the time dependency in the differential Harnack estimate.

MSC:

35R02 PDEs on graphs and networks (ramified or polygonal spaces)
35B45 A priori estimates in context of PDEs
35K05 Heat equation
35K10 Second-order parabolic equations
05C10 Planar graphs; geometric and topological aspects of graph theory
05C81 Random walks on graphs

Keywords:

differential Harnack inequality

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