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Solving the discrete Euler-Arnold equations for the generalized rigid body motion. (English)

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Summary: We propose three iterative methods for solving the Moser-Veselov equation, which arises in the discretization of the Euler-Arnold differential equations governing the motion of a generalized rigid body. We start by formulating the problem as an optimization problem with orthogonal constraints and proving that the objective function is convex. Then, using techniques from optimization on Riemannian manifolds, the three feasible algorithms are designed. The first one splits the orthogonal constraints using the Bregman method, whereas the other two methods are of the steepest-descent type. The second method uses the Cayley-transform to preserve the constraints and a Barzilai-Borwein step size, while the third one involves geodesics, with the step size computed by Armijo's rule. Finally, a set of numerical experiments is carried out to compare the performance of the proposed algorithms, suggesting that the first algorithm has the best performance in terms of accuracy and number of iterations. An essential advantage of these iterative methods is that they work even when the conditions for applicability of the direct methods available in the literature are not satisfied.

MSC:

90C30 Nonlinear programming

90C90 Applications of mathematical programming

Keywords:

discrete Euler-Arnold equations; matrix equation; Moser-Veselov equation; optimization with orthogonal constraints; orthogonal matrices; skew-symmetric matrices

Software:

mfttoolbox

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References:

- [1] Moser, J.; Veselov, A. P., Discrete versions of some classical integrable systems and factorization of matrix polynomials, *Comm. Math. Phys.*, 139, 2, 217-243 (1991) · [Zbl 0754.58017](#)
- [2] Bloch, A., *Nonholonomic Mechanics and Control* (2015), Springer
- [3] Dang, Q.; Gui, H.; Liu, K.; Zhu, B., Relaxed-constraint pinpoint lunar landing using geometric mechanics and model predictive control, *J. Guid. Control Dyn.* (2020), URL <https://doi.org/10.2514/1.G005039>
- [4] Lee, T.; Leok, M.; McClamroch, N., Lie group variational integrators for the full body problem in orbital mechanics, *Celestial Mech. Dynam. Astronom.*, 98, 121-144 (2007) · [Zbl 1330.70003](#)
- [5] Nordkvist, N.; Sanyal, A., A Lie group variational integrator for rigid body motion in $se(3)$ with applications to underwater vehicle dynamics, (*IEEE Conference on Decision and Control (CDC)* (2010)), 5414-5419
- [6] Kalabic, U.; Gupta, R.; Cairano, S. D.; Bloch, A. M.; Kolmanovsky, I. V., Constrained spacecraft attitude control on $SO(3)$ using reference governors and nonlinear model predictive control, (*American Control Conference* (2014)), 5586-5593
- [7] Kalabic, U.; Gupta, R.; Cairano, S. D.; Bloch, A. M.; Kolmanovsky, I. V., MPC on manifolds with an application to the control of spacecraft attitude on $SO(3)$, *Automatica*, 76, 293-300 (2017) · [Zbl 1352.93035](#)
- [8] Cardoso, J.; Leite, F., The moser-veselov equation, *Linear Algebra Appl.*, 360, 237-248 (2003) · [Zbl 1020.15016](#)
- [9] McLachlan, R.; Zanna, A., The discrete moser—Veselov algorithm for the free rigid body, revisited, *Found. Comput. Math.*, 5, 87-123 (2005) · [Zbl 1123.70012](#)
- [10] Nocedal, J.; Wright, S., *Numerical Optimization* (2006), Springer: Springer New York, NY, USA · [Zbl 1104.65059](#)
- [11] Luenberger, D.; Ye, Y., *Linear and Nonlinear Programming* (2015), Springer Publishing Company, Incorporated
- [12] Polak, E., *Optimization: Algorithms and Consistent Approximations* (1997), Springer-Verlag · [Zbl 0899.90148](#)
- [13] Miraldo, P.; Cardoso, J. R., On the generalized essential matrix correction: An efficient solution to the problem and its

- applications, *J. Math. Imaging Vision*, 62, 1107-1120 (2020) · [Zbl 07289204](#)
- [14] Campos, J.; Cardoso, J. R.; Miraldo, P., POSEAMM: A unified framework for solving pose problems using an alternating minimization method, (*IEEE Int'L Conf. Robotics and Automation (ICRA)* (2019)), 3493-3499
- [15] Absil, P.-A.; Mahony, R.; Sepulchre, R., *Optimization Algorithms on Matrix Manifolds* (2007), Princeton University Press · [Zbl 1147.65043](#)
- [16] Edelman, A.; Arias, T.; Smith, S., The geometry of algorithms with orthogonality constraints, *SIAM J. Matrix Anal. Appl.*, 20, 2, 303-353 (1999) · [Zbl 0928.65050](#)
- [17] Lutkepohl, H., *Handbook of Matrices* (1996), John Wiley and Sons · [Zbl 0856.15001](#)
- [18] Abrudan, T.; Eriksson, J.; Koivunen, V., Steepest descent algorithms for optimization under unitary matrix constraint, *Trans. Signal Process.*, 56, 1134-1147 (2008) · [Zbl 1390.90510](#)
- [19] Gao, B.; Liu, X.; Chen, X.; Yuan, Y.-X., A new first-order algorithmic framework for optimization problems with orthogonality constraints, *SIAM J. Optim.*, 28, 302-332 (2018) · [Zbl 1382.65171](#)
- [20] Jiang, B.; Dai, Y.-H., A framework of constraint preserving update schemes for optimization on Stiefel manifold, *Math. Program.*, 153, 535-575 (2015) · [Zbl 1325.49037](#)
- [21] Lai, R.; Osher, S., A splitting method for orthogonality constrained problems, *J. Sci. Comput.*, 58, 431-449 (2014) · [Zbl 1296.65087](#)
- [22] Manton, J., Optimization algorithms exploiting unitary constraints, *Trans. Signal Process.*, 50, 635-650 (2002) · [Zbl 1369.90169](#)
- [23] Wen, Z.; Yin, W., A feasible method for optimization with orthogonality constraints, *Math. Program.*, 142, 397-434 (2013) · [Zbl 1281.49030](#)
- [24] Zhu, X., A Riemannian conjugate gradient method for optimization on the Stiefel manifold, *Comput. Optim. Appl.*, 67, 73-110 (2017) · [Zbl 1401.90230](#)
- [25] Bregman, L., The relaxation method of finding the common points of convex sets and its application to the solution of problems in convex optimization, *USSR Comput. Math. Math. Phys.*, 7, 200-217 (1967)
- [26] Osher, S.; Burger, M.; Goldfarb, D.; Xu, J.; Yin, W., An iterative regularization method for total variation-based image restoration, *Multiscale Model. Simul.*, 4, 460-489 (2005) · [Zbl 1090.94003](#)
- [27] Barzilai, J.; Borwein, J., Two-point step size gradient methods, *IMA J. Numer. Anal.*, 8, 141-148 (1988) · [Zbl 0638.65055](#)
- [28] Coleman, T.; Li, Y., On the convergence of reflective Newton methods for large-scale nonlinear minimization subject to bounds, *Math. Program.*, 67, 2, 189-224 (1994) · [Zbl 0842.90106](#)
- [29] Coleman, T.; Li, Y., An interior, trust region approach for nonlinear minimization subject to bounds, *SIAM J. Optim.*, 6, 418-445 (1996) · [Zbl 0855.65063](#)
- [30] Terán, F. D.; Dopico, F., Consistency and efficient solution of the Sylvester equation for (A, X) -congruence: $(A X + X^* B = C)$, *Electron. J. Linear Algebra*, 22, 849-863 (2011) · [Zbl 1256.65035](#)
- [31] Dai, Y.-H.; Fletcher, R., Projected Barzilai-Borwein methods for large-scale box-constrained quadratic programming, *Numer. Math.*, 100, 21-47 (2005) · [Zbl 1068.65073](#)
- [32] Golub, G.; Reinsch, C., Singular value decomposition and least squares solutions, *Numer. Math.*, 14, 403-420 (1970) · [Zbl 0181.17602](#)
- [33] Cardoso, J.; Leite, F., Exponentials of skew-symmetric matrices and logarithms of orthogonal matrices, *J. Comput. Appl. Math.*, 233, 2867-2875 (2010) · [Zbl 1185.65070](#)
- [34] Al-Mohy, A.; Higham, N., A new scaling and squaring algorithm for the matrix exponential, *SIAM J. Matrix Anal. Appl.*, 31, 970-989 (2009) · [Zbl 1194.15021](#)
- [35] Guo, C.-H.; Higham, N., A Schur-Newton method for the matrix p-th root and its inverse, *SIAM J. Matrix Anal. Appl.*, 28, 788-804 (2006) · [Zbl 1128.65030](#)
- [36] Higham, N., *Functions of Matrices: Theory and Computation* (2008), SIAM: SIAM Philadelphia, PA, USA

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