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Superconvergent HDG methods for Maxwell's equations via the M-decomposition. (English)

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Summary: The concept of the M-decomposition was introduced by Cockburn et al. to provide criteria to guarantee optimal convergence rates for the Hybridizable Discontinuous Galerkin (HDG) method for coercive elliptic problems. In that paper they systematically constructed superconvergent hybridizable discontinuous Galerkin (HDG) methods to approximate the solutions of elliptic PDEs on unstructured meshes. In this paper, we use the M-decomposition to construct HDG methods for the Maxwell's equations on unstructured meshes in two dimension. In particular, we show the any choice of spaces having an M-decomposition, together with sufficiently rich auxiliary spaces, has an optimal error estimate and superconvergence even though the problem is not in general coercive. Motivated by the elliptic case, we obtain a superconvergent rate for the curl and flux of the solution, and this is confirmed by our numerical experiments.

MSC:

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
65N12 Stability and convergence of numerical methods for boundary value problems involving PDEs
65N15 Error bounds for boundary value problems involving PDEs
78A25 Electromagnetic theory (general)
78M10 Finite element, Galerkin and related methods applied to problems in optics and electromagnetic theory
35Q61 Maxwell equations

Keywords:

Maxwell's equations; M-decomposition; HDG method; error analysis

Software:

iHDG

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