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Duan, Huoyuan; **Ma, Junhua**; **Tan, Roger C. E.**; **Wang, Can**

A coercive mixed formulation for the generalized Maxwell problem. (English) Zbl 1477.65208

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Summary: A coercive mixed variational formulation on $H_0(\mathbf{curl};\Omega)\times H(\mathrm{div};\Omega)$ is proposed for the generalized Maxwell problem which typically arises from computational electromagnetism. The mixed variables are the electric field and a pseudo electric displacement field. The well-posedness of the mixed variational problem is proven in the general settings (multiply connected domain of Lipschitz-continuous boundary with a number of connected components, filling with discontinuous, anisotropic and inhomogeneous media); more importantly, the coercivity is established. A conforming finite element discretization is further proposed, where the electric field is approximated by $H(\mathbf{curl};\Omega)$-conforming edge element while the pseudo electric displacement field by $H(\mathrm{div};\Omega)$-conforming flux element. Error estimates are obtained, and in particular, the method produces an L^2 curl-convergent approximation and more importantly, an L^2 div-convergent approximation for the solution.

MSC:

65N30	Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
65N12	Stability and convergence of numerical methods for boundary value problems involving PDEs
65N15	Error bounds for boundary value problems involving PDEs
35A01	Existence problems for PDEs: global existence, local existence, non-existence
35A02	Uniqueness problems for PDEs: global uniqueness, local uniqueness, non-uniqueness
78A25	Electromagnetic theory (general)
78M10	Finite element, Galerkin and related methods applied to problems in optics and electromagnetic theory
35Q61	Maxwell equations

Keywords:

Maxwell problem; mixed formulation; edge element; flux element; coercivity; L^2 div-convergence

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