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Nonexistence of solutions for indefinite fractional parabolic equations. (English)

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Summary: We study fractional parabolic equations with indefinite nonlinearities

$$\frac{\partial u}{\partial t}(x, t) + (-\Delta)^s u(x, t) = x_1 u^p(x, t), \quad (x, t) \in \mathbb{R}^n \times \mathbb{R},$$

where $0 < s < 1$ and $1 < p < \infty$. We first prove that all positive bounded solutions are monotone increasing along the x_1 direction. Based on this we derive a contradiction and hence obtain non-existence of solutions. These monotonicity and nonexistence results are crucial tools in a priori estimates and complete blow-up for fractional parabolic equations in bounded domains. To this end, we introduce several new ideas and developed a systematic approach which may also be applied to investigate qualitative properties of solutions for many other fractional parabolic problems.

MSC:

[35B53](#) Liouville theorems and Phragmén-Lindelöf theorems in context of PDEs

[35R11](#) Fractional partial differential equations

[30C80](#) Maximum principle, Schwarz's lemma, Lindelöf principle, analogues and generalizations; subordination

[35K15](#) Initial value problems for second-order parabolic equations

[35K58](#) Semilinear parabolic equations

Keywords:

[Liouville theorem](#); [fractional parabolic equation](#); [indefinite nonlinearity](#); [monotonicity](#); [method of moving planes](#)

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