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Diffusive effects in local instabilities of a baroclinic axisymmetric vortex. (English)

Zbl 1481.76101
J. Fluid Mech. 928, Paper No. A14, 32 p. (2021).

Summary: We present a local stability analysis of an idealized model of the stratified vortices that appear in geophysical settings. The base flow comprises an axisymmetric vortex with background rotation and an out-of-plane stable stratification, and a radial stratification in the thermal wind balance with the out-of-plane momentum gradient. Solving the local stability equations along fluid particle trajectories in the base flow, the dependence of short-wavelength instabilities on the Schmidt number Sc (ratio between momentum and mass diffusivities) is studied, in the presence of curvature effects. In the diffusion-free limit, the well-known symmetric instability is recovered. In the viscous, double-diffusive regime, instability characteristics are shown to depend on three non-dimensional parameters (including Sc), and two different instabilities are identified: (i) a monotonic instability (same as symmetric instability at $Sc = 1$), and (ii) an oscillatory instability (absent at $Sc = 1$). Separating the base flow and perturbation characteristics, two each of base flow and perturbation parameters (apart from Sc) are identified, and the entire parameter space is explored for the aforementioned instabilities. In comparison with $Sc = 1$, monotonic and oscillatory instabilities are shown to significantly expand the instability region in the space of base flow parameters as Sc moves away from unity. Neutral stability boundaries on the plane of Sc and a modified gradient Richardson number are then identified for both these instabilities. In the absence of curvature effects, our results are shown to be consistent with previous studies based on normal mode analysis, thus establishing that the local stability approach is well suited to capturing symmetric and double-diffusive instabilities. The paper concludes with a discussion of curvature effects, and the likelihood of monotonic and oscillatory instabilities in typical oceanic settings.

MSC:

76E20 Stability and instability of geophysical and astrophysical flows
76E07 Rotation in hydrodynamic stability
76D17 Viscous vortex flows
76R50 Diffusion
76U60 Geophysical flows

Keywords:

baroclinic rotating flow; vortex instability; viscous double-diffusive regime; Floquet theory

Full Text: DOI

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.