

Sahoo, Sudeep K.; Toth, Laszlo S.; Molinari, Alain; Latypov, Marat I.; Bouaziz, Olivier
Plastic energy-based analytical approach to predict the mechanical response of two-phase materials with application to dual-phase steels. (English) [Zbl 1479.74027](#)
Eur. J. Mech., A, Solids 91, Article ID 104414, 15 p. (2022).

Summary: A composite made of two phases is considered with a perfect disorder of the phases, and isotropic behavior. The strain hardening behavior of such a composite is modeled under axisymmetric tension. The approach is based on using the strain hardening behavior of the two constituent phases together with a relation between the plastic energy of the two phases. The newly developed analytical model was applied to several dual-phase steel alloys and on iron-silver composite metal. These case studies revealed that the equal-power approach reproduces faithfully the strain hardening behavior of the composite, together with the strain partitioning between the two phases, in good agreement with experiments.

MSC:

- 74E30 Composite and mixture properties
- 74C99 Plastic materials, materials of stress-rate and internal-variable type
- 74S05 Finite element methods applied to problems in solid mechanics

Keywords:

homogenization; plastic energy; strain hardening; strain partition; composite model; dual-phase steel; finite element computation

Full Text: [DOI](#)

References:

- [1] Aboudi, J.; Pindera, M.-J., *Matrix Mean-Field and Local-Field Approaches in the Analysis of Metal Matrix Composites*, 761-779 (1991), Springer New York: Springer New York New York, NY, doi.org/
- [2] Berbenni, S.; Capolungo, L., A Mori-Tanaka homogenization scheme for non-linear elasto-viscoplastic heterogeneous materials based on translated fields: an affine extension, *CRM*, 343, 2, 95-106 (2015)
- [3] Berveiller, M.; Zaoui, A., An extension of the self-consistent scheme to plastically-flowing polycrystals, *J. Mech. Phys. Solid.*, 26, 5-6, 325-344 (1979) · [Zbl 0395.73033](#)
- [4] Bornert, M.; Hervé, E.; Stolz, C.; Zaoui, A., Self-consistent approaches and strain heterogeneities in two-phase elastoplastic materials, *Appl. Mech. Rev.*, 47, 1S, S66 (1994)
- [5] Bosco, E.; Kouznetsova, V. G.; Coenen, E. W.C.; Geers, M. G.D.; Salvadori, A., A multiscale framework for localizing microstructures towards the onset of macroscopic discontinuity, *Comput. Mech.*, 54, 2, 299-319 (2014) · [Zbl 1398.74308](#)
- [6] Bouaziz, O.; Buessler, P., Mechanical behaviour of multiphase materials: an intermediate mixture law without fitting parameter, *Rev. Métall.*, 99, 1, 71-77 (2002)
- [7] Bouaziz, O.; Buessler, P., Iso-work increment assumption for heterogeneous material behaviour modelling, *Adv. Eng. Mater.*, 6, 12, 79-83 (2004)
- [8] Brands, D.; Balzani, D.; Scheunemann, L.; Schröder, J.; Richter, H.; Raabe, D., Computational modeling of dual-phase steels based on representative three-dimensional microstructures obtained from EBSD data, *Arch. Appl. Mech.*, 86, 3, 575-598 (2015)
- [9] Brassart, L.; Doghri, I.; Delannay, L., Homogenization of elasto-plastic composites coupled with a nonlinear finite element analysis of the equivalent inclusion problem, *Int. J. Solid Struct.*, 47, 5, 716-729 (2010) · [Zbl 1183.74044](#)
- [10] Brassart, L.; Stainier, L.; Doghri, I.; Delannay, L., Homogenization of elasto-(visco) plastic composites based on an incremental variational principle, *Int. J. Plast.*, 36, 86-112 (2012)
- [11] Brenner, R.; Castelnau, O.; Gilormini, P., A modified affine theory for the overall properties of nonlinear composites, *CRAS (Series IIB)*, 329, 9, 649-654 (2001) · [Zbl 1027.74057](#)
- [12] Brenner, R.; Masson, R., Improved affine estimates for nonlinear viscoelastic composites, *Eur. J. Mech. A Solids*, 24, 6, 1002-1015 (2005) · [Zbl 1098.74663](#)
- [13] Budiansky, B., On the elastic moduli of some heterogeneous materials, *J. Mech. Phys. Solid.*, 13, 4, 223-227 (1965)
- [14] Calvo-Jurado, C.; Parnell, W. J., The influence of two-point statistics on the Hashin-Shtrikman bounds for three phase composites, *J. Comput. Appl. Math.*, 318, 354-365 (2017) · [Zbl 1366.74065](#)

- [15] Chandiran, E.; Sato, Y.; Kamikawa, N.; Miyamoto, G.; Furuhashi, T., Effect of ferrite/martensite phase size on tensile behavior of dual-phase steels with nano-precipitation of vanadium carbides, *Metall. Mater. Trans. A* 50A, 4111-4126 (2019)
- [16] Delannay, L.; Doghri, I.; Pierard, O., Prediction of tension-compression cycles in multiphase steel using a modified incremental mean-field model, *Int. J. Solid Struct.*, 44, 22-23, 7291-7306 (2007) · [Zbl 1166.74410](#)
- [17] Delannay, L.; Jacques, P.; Pardoën, T., Modelling of the plastic flow of trip-aided multiphase steel based on an incremental mean-field approach, *Int. J. Solid Struct.*, 45, 6, 1825-1843 (2008) · [Zbl 1159.74331](#)
- [18] Dhinwal, S. S.; Toth, L. S., Unlocking deformation path in asymmetric rolling by texture simulation, *Materials*, 13, 1, 101 (2019)
- [19] Doghri, I.; Adam, L.; Bilger, N., Mean-field homogenization of elasto-viscoplastic composites based on a general incrementally affine linearization method, *Int. J. Plast.*, 26, 2, 219-238 (2010) · [Zbl 1426.74111](#)
- [20] Doghri, I.; Brassart, L.; Adam, L.; Gérard, J.-S., A second-moment incremental formulation for the mean-field homogenization of elasto-plastic composites, *Int. J. Plast.*, 27, 3, 352-371 (2011) · [Zbl 1426.74112](#)
- [21] Doghri, I.; El Ghezal, M. I.; Adam, L., Finite strain mean-field homogenization of composite materials with hyperelastic-plastic constituents, *Int. J. Plast.*, 81, 40-62 (2016)
- [22] Doghri, I.; Ouaar, A., Homogenization of two-phase elasto-plastic composite materials and structures, *Int. J. Solid Struct.*, 40, 7, 1681-1712 (2003) · [Zbl 1032.74624](#)
- [23] Eshelby, J. D., The determination of the elastic field of an ellipsoidal inclusion, and related problems, *Proc. Roy. Soc. Lond. A*, 241, 1226, 376-396 (1957) · [Zbl 0079.39606](#)
- [24] Geers, M. G.D.; Kouznetsova, V. G.; Brekelmans, W. A.M., Multi-scale computational homogenization: trends and challenges, *J. Comput. Appl. Math.*, 234, 7, 2175-2182 (2010) · [Zbl 1402.74107](#)
- [25] Gilormini, P., Insuffisance de l'extension classique du modèle autocohérent au comportement non linéaire, *C. R. Acad. Sci.*, 314, Series II, 115-122 (1995) · [Zbl 0834.73013](#)
- [26] Gilormini, P.; Germain, Y., A finite element analysis of the inclusion problem for power law viscous materials, *Int. J. Solid Struct.*, 23, 3, 413-437 (1987) · [Zbl 0605.73012](#)
- [27] Hashin, Z.; Shtrikman, S., A variational approach to the theory of the elastic behaviour of multiphase materials, *J. Mech. Phys. Solid.*, 11, 2, 127-140 (1963) · [Zbl 0108.36902](#)
- [28] Hershey, A., The elasticity of an isotropic aggregate of anisotropic cubic crystals, *J. Appl. Mech.*, 21, 3, 236-240 (1954) · [Zbl 0059.17604](#)
- [29] Hill, R., A self-consistent mechanics of composite materials, *J. Mech. Phys. Solid.*, 13, 4, 213-222 (1965)
- [30] Hutchinson, J. W., Elastic-plastic behaviour of polycrystalline metals and composites, *Proc. Roy. Soc. Lond. A*, 319, 1537, 247-272 (1970)
- [31] Idiart, M. I.; Moulinec, H.; Ponte Castañeda, P.; Suquet, P., Macroscopic behavior and field fluctuations in viscoplastic composites: second-order estimates versus full-field simulations, *J. Mech. Phys. Solid.*, 54, 5, 1029-1063 (2006) · [Zbl 1120.74720](#)
- [32] Ishikawa, N.; Yasuda, K.; Sueyoshi, H.; Endo, S.; Ikeda, H.; Morikawa, T.; Higashida, K., Microscopic deformation and strain hardening analysis of ferrite-bainite dual-phase steels using micro-grid method, *Acta Mater.*, 97, 257-268 (2015)
- [33] Ismail, K.; Perlade, A.; Jacques, P. P.; Pardoën, T.; Brassart, L., Impact of second phase morphology and orientation on the plastic behavior of dual-phase steels, *IJP*, 118, 130-146 (2019), [org/10.1016/j.ijplas.2019.02.005](https://doi.org/10.1016/j.ijplas.2019.02.005)
- [34] Kadkhodapour, J.; Butz, A.; Ziaei-Rad, S.; Schmauder, S., A micro mechanical study on failure initiation of dual phase steels under tension using single crystal plasticity model, *Int. J. Plast.*, 27, 7, 1103-1125 (2011) · [Zbl 1441.74225](#)
- [35] Kalidindi, S. R.; Bronkhorst, C. A.; Anand, L., Crystallographic texture evolution in bulk deformation processing of FCC metals, *J. Mech. Phys. Solid.*, 40, 3, 537-569 (1992)
- [36] Kamikawa, N.; Hirohashi, M.; Sato, Y.; Chandiran, E.; Miyamoto, G.; Furuhashi, T., Tensile behavior of ferrite-martensite dual phase steels with nano-precipitation of vanadium carbides, *ISIJ Int.*, 55, 8, 1781-1790 (2015)
- [37] Kanaun, S. K.; Levin, V. M., Self-Consistent Methods for Composites, *Solid Mechanics and its Applications* (2008) · [Zbl 1142.74003](#)
- [38] Kanouté, P.; Boso, D. P.; Chaboche, J. L.; Schrefler, B. A., Multiscale methods for composites: a review, *Arch. Comput. Methods Eng.*, 16, 1, 31-75 (2009) · [Zbl 1170.74304](#)
- [39] Kim, D.-K.; Kim, E.-Y.; Han, J.; Woo, W.; Choi, S.-H., Effect of microstructural factors on void formation by ferrite/martensite interface decohesion in DP980 steel under uniaxial tension, *Int. J. Plast.*, 94, 3-23 (2017)
- [40] Kreher, W., Residual stresses and stored elastic energy of composites and polycrystals, *J. Mech. Phys. Solid.*, 38, 1, 115-128 (1990) · [Zbl 0701.73008](#)
- [41] Kreher, W.; Molinari, A., Residual stresses in polycrystals as influenced by grain shape and texture, *J. Mech. Phys. Solid.*, 41, 12, 1955-1977 (1993) · [Zbl 0792.73069](#)
- [42] Kröner, E., Zur plastischen verformung des vielkristalls, *Acta Metall.*, 9, 2, 155-161 (1961)
- [43] Kröner, E., Bounds for effective elastic moduli of disordered materials, *J. Mech. Phys. Solid.*, 25, 2, 137-155 (1977) · [Zbl 0359.73020](#)
- [44] Kröner, E., Self-consistent scheme and graded disorder in polycrystal elasticity, *J. Phys. F Met. Phys.*, 8, 11, 2261-2267 (1978)
- [45] Lahellec, N.; Suquet, P., Effective response and field statistics in elasto-plastic and elasto-viscoplastic composites under radial and non-radial loadings, *Int. J. Plast.*, 42, 1-30 (2013)

- [46] Lai, Q.; Brassart, L.; Bouaziz, O.; Gouné, M.; Verdier, M.; Parry, G.; Perlade, A.; Bréchet, A.; Pardoën, T., Influence of martensite volume fraction and hardness on the plastic behavior of dual-phase steels: experiments and micromechanical modeling, *Int. J. Plast.*, 80, 187-203 (2016)
- [47] Latypov, M. I.; Shin, S.; De Cooman, B. C.; Kim, H. S., Micromechanical finite element analysis of strain partitioning in multiphase medium manganese TWIP+TRIP steel, *Acta Mater.*, 108, 219-228 (2016), doi:10.1016/s0022-5096(02)00021-2
- [48] Latypov, M. I.; Toth, L. S.; Kalidindi, S. R., Materials knowledge system for nonlinear composites, *Comput. Methods Appl. Mech. Eng.*, 346, 180-196 (2018) · [Zbl 1440.74105](#)
- [49] Lebensohn, R. A.; Tomé, C. N., A self-consistent anisotropic approach for the simulation of plastic deformation and texture development of polycrystals: application to zirconium alloys, *Acta Metall.*, 41, 9, 2611-2624 (1993)
- [50] Lebensohn, R. A.; Tomé, C. N.; Ponte Castañeda, P., Self-consistent modelling of the mechanical behaviour of viscoplastic polycrystals incorporating intragranular field fluctuations, *Phil. Mag.*, 87, 28, 4287-4322 (2007)
- [51] Liang, X.; McDermid, J. R.; Bouaziz, O.; Wang, X.; Embury, J. D.; Zurob, H. S., Microstructural evolution and strain hardening of Fe-24Mn and Fe-30Mn alloys during tensile deformation, *Acta Mater.*, 57, 13, 3978-3988 (2009)
- [52] Masson, R.; Zaoui, A., Self-consistent estimates for the rate-dependent elastoplastic behaviour of polycrystalline materials, *J. Mech. Phys. Solid.*, 47, 7, 1543-1568 (1999) · [Zbl 0976.74010](#)
- [53] Mercier, S.; Molinari, A., Homogenization of elastic-viscoplastic heterogeneous materials: self-consistent and Mori-Tanaka schemes, *Int. J. Plast.*, 25, 6, 1024-1048 (2009) · [Zbl 1162.74037](#)
- [54] Molinari, A.; El Houdaigui, F.; Tóth, L. S., Validation of the tangent formulation for the solution of the non-linear Eshelby inclusion problem, *Int. J. Plast.*, 20, 2, 291-307 (2004) · [Zbl 1145.74328](#)
- [55] Molinari, A.; Tóth, L. S., Tuning a self consistent viscoplastic model by finite element results-I. Modeling, *Acta Metall.*, 42, 7, 2453-2458 (1994)
- [56] Molinari, A.; Canova, G. R.; Ahzi, S., A self consistent approach of the large deformation polycrystal viscoplasticity, *Acta Metall.*, 35, 12, 2983-2994 (1987)
- [57] Montheillet, F.; Damamme, G., Simple flow rules for modeling the behaviour of inhomogeneous viscoplastic materials, *Adv. Eng. Mater.*, 7, 9, 852-858 (2005)
- [58] Mori, T.; Tanaka, K., Average stress in matrix and average elastic energy of materials with misfitting inclusions, *Acta Metall.*, 21, 5, 571-574 (1973)
- [59] Moulinec, H.; Suquet, P., Intra-phase strain heterogeneity in nonlinear composites: a computational approach, *Eur. J. Mech. A Solids*, 22, 5, 751-770 (2003) · [Zbl 1032.74508](#)
- [60] Nguyen-Thanh, V. M.; Nguyen, L. T.K.; Rabczuk, T.; Zhuang, X., A surrogate model for computational homogenization of elastostatics at finite strain using HDMMR -based neural network, *Int. J. Numer. Methods Eng.* (2020)
- [61] Ortiz, M.; Molinari, A., Microstructural thermal stresses in ceramic materials, *J. Mech. Phys. Solid.*, 36, 4, 385-400 (1988) · [Zbl 0637.73008](#)
- [62] Ortolano, J. M.; Hernández, J. A.; Oliver, J., A comparative study on homogenization strategies for multi-scale analysis of materials, *Monograph CIMNE*, 135 (2013)
- [63] Peng, X.; Hu, N.; Long, X.; Zheng, H., Extension of combined self-consistent and Mori-Tanaka approach to evaluation of elastoplastic property of particulate composites, *Acta Mech. Solida Sin.*, 26, 1, 71-82 (2013)
- [64] Peng, X.; Hu, N.; Zheng, H.; Fukunaga, H., Evaluation of mechanical properties of particulate composites with a combined self-consistent and Mori-Tanaka approach, *Mech. Mater.*, 41, 12, 1288-1297 (2009)
- [65] Peng, X.; Tang, S.; Hu, N.; Han, J., Determination of the Eshelby tensor in mean-field schemes for evaluation of mechanical properties of elastoplastic composites, *Int. J. Plast.*, 76, 147-165 (2016)
- [66] Pierard, O.; Doghri, I., Study of various estimates of the macroscopic tangent operator in the incremental homogenization of elastoplastic composites, *Int. J. Multiscale Comput. Eng.*, 4, 521-543 (2006)
- [67] Pierard, O.; Llorca, J.; Segurado, J.; Doghri, I., Micromechanics of particle-reinforced elasto-viscoplastic composites: finite element simulations versus affine homogenization, *Int. J. Plast.*, 23, 6, 1041-1060 (2007) · [Zbl 1119.74014](#)
- [68] Ponte-Castañeda, P., The effective mechanical properties of nonlinear isotropic composites, *J. Mech. Phys. Solid.*, 39, 1, 45-71 (1991) · [Zbl 0734.73052](#)
- [69] Ponte-Castañeda, P., Second-order homogenization estimates for nonlinear composites incorporating field fluctuations: I-theory, *J. Mech. Phys. Solid.*, 50, 4, 737-757 (2002) · [Zbl 1116.74412](#)
- [70] Ponte Castañeda, P.; Suquet, P., Nonlinear composites, *Adv. Appl. Mech.*, 34, 171-302 (1997) · [Zbl 0889.73049](#)
- [71] Ponte-Castañeda, P.; Willis, J. R., On the overall properties of nonlinearly viscous composites, *Proc. Roy. Soc. Lond. A*, 416, 1850, 217-244 (1988) · [Zbl 0635.73006](#)
- [72] Ramazani, A.; Mukherjee, K.; Quade, H.; Prah, U.; Bleck, W., Correlation between 2D and 3D flow curve modelling of DP steels using a microstructure-based RVE approach, *Mater. Sci. Eng., A*, 560, 129-139 (2013)
- [73] Ramazani, A.; Pinard, P. T.; Richter, S.; Schwedt, A.; Prah, U., Characterisation of microstructure and modelling of flow behaviour of bainite-aided dual-phase steel, *Comput. Mater. Sci.*, 80, 134-141 (2013)
- [74] Reuss, A., Berechnung der Fließgrenze von Mischkristallen auf Grund der Plastizitätsbedingung für Einkristalle, *ZAMM - J. Appl. Math. Mech./Z. Angew. Math. Mech.*, 9, 49-58 (1929) · [Zbl 55.1110.02](#)
- [75] Sahoo, S. K.; Biswas, S.; Toth, L. S.; Gautam, P. C.; Beausir, B., Strain hardening, twinning and texture evolution in magnesium alloy using the all twin variant polycrystal modelling approach, *Int. J. Plast.*, 128, 1-30 (2020)

- [76] Sahoo, S. K.; Toth, L. S.; Biswas, S., An analytical model to predict strain-hardening behaviour and twin volume fraction in a profoundly twinning magnesium alloy, *Int. J. Plast.*, 119, 273-290 (2019)
- [77] Sahoo, S. K.; Dhinwal, S. S.; Vu, V. Q.; Toth, L. S., A new macroscopic strain hardening function based on microscale crystal plasticity, *Mater. Sci. Eng., A*, 141634 (2021)
- [78] Song, Z.; Peng, X.; Tang, S.; Fu, T., A homogenization scheme for elastoplastic composites using concept of Mori-Tanaka method and average deformation power rate density, *Int. J. Plast.*, 102652 (2020)
- [79] Stringfellow, R. G.; Parks, D. M., A self-consistent model of isotropic viscoplastic behavior in multiphase materials, *Int. J. Plast.*, 7, 6, 529-547 (1991) · [Zbl 0825.73192](#)
- [80] Sun, X.; Choi, K. S.; Liu, W. N.; Khaleel, M. A., Predicting failure modes and ductility of dual phase steels using plastic strain localization, *Int. J. Plast.*, 25, 10, 1888-1909 (2009)
- [81] Suquet, P., Overall properties of nonlinear composites: a modified secant moduli theory and its link with Ponte Castañeda's nonlinear variational procedure, *C. R. Acad. Sci. Paris Sér. IIB*, 320, 563-571 (1995) · [Zbl 0830.73046](#)
- [82] Suquet, P., Effective properties of nonlinear composites, (Suquet, P., *Continuum Micromechanics*, CISM Lecture Notes, vol. 377 (1997), Springer Verlag: Springer Verlag New York), 197-264 · [Zbl 0883.73051](#)
- [83] Talbot, D. R.S.; Willis, J. R., Variational principles for inhomogeneous non-linear media, *IMA J. Appl. Math.*, 35, 1, 39-54 (1985) · [Zbl 0588.73025](#)
- [84] Tasan, C. C.; Hoefnagels, J. P.M.; Diehl, M.; Yan, D.; Roters, F.; Raabe, D., Strain localization and damage in dual phase steels investigated by coupled in-situ deformation experiments and crystal plasticity simulations, *Int. J. Plast.*, 63, 198-210 (2014)
- [85] Tandon, G. P.; Weng, G. J., A theory of particle-reinforced plasticity, *J. Appl. Mech.*, 55, 1, 126-135 (1988)
- [86] Torquato, S., Random heterogeneous media: microstructure and improved bounds on effective properties, *Appl. Mech. Rev.*, 44, 2, 37 (1991)
- [87] Tóth, L. S.; Molinari, A.; Bouaziz, O., Effective strain rate sensitivity of two phase materials, *Mater. Sci. Eng., A*, 524, 1-2, 186-192 (2009)
- [88] Turner, P. A.; Tomé, C. N., A study of residual stresses in Zircaloy-2 with rod texture, *Acta Metall. Mater.*, 42, 12, 4143-4153 (1994)
- [89] Voigt, W., *Handbook of Crystal Physics* (1928), Teubner: Teubner Leipzig, Germany
- [90] Willis, J. R., Bounds and self-consistent estimates for the overall properties of anisotropic composites, *J. Mech. Phys. Solid.*, 25, 3, 185-202 (1977) · [Zbl 0363.73014](#)
- [91] Wu, L.; Noels, L.; Adam, L.; Doghri, I., A combined incremental-secant mean-field homogenization scheme with per-phase residual strains for elasto-plastic composites, *Int. J. Plast.*, 51, 80-102 (2013)
- [92] Wu, L.; Doghri, I.; Noels, L., An incremental-secant mean-field homogenization method with second statistical moments for elasto-plastic composite materials, *Phil. Mag.*, 95, 28-30, 3348-3384 (2015)
- [93] Zhou, Y.; Neale, K. W.; Tóth, L. S., A modified model for simulating latent hardening during the plastic deformation of rate-dependent FCC polycrystals, *Int. J. Plast.*, 9, 8, 961-978 (1993)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.