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Decay character and estimates for the damped wave equation. (English) Zbl 1480.35030
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The study refers to the linear damped wave equation $u_{tt} - \Delta u + u_t = 0$, $u(0) = f$, $u_t(0) = g$, where $u = u(x, t)$, $(x, t) \in \mathbb{R}^n \times [0, \infty)$, as well as to the damped wave equation with absorption, $u_{tt} - \Delta u + u_t = -|u|^\alpha u$, $u(0) = f$, $u_t(0) = g$. The aim is to investigate decay properties of solutions and improve some known estimates on solutions and energy. Addressed are mainly the earlier results of *A. Matsumura* [Publ. Res. Inst. Math. Sci. 12, 169–189 (1976; Zbl 0356.35008)], and the results of *R. Ikehata* [Math. Methods Appl. Sci. 27, No. 8, 865–889 (2004; Zbl 1049.35135)]. The first section of the article is devoted to the presentation of the known estimates and decay results as well as to the presentation of new results. In the second section definitions and properties connected to the decay of solutions and properties of fundamental solutions to the linear damped equation are reviewed. The proofs of new estimates and decay of solutions are to be found in the last part of the second section and in the third section.

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MSC:

- [35B40](#) Asymptotic behavior of solutions to PDEs
- [35L15](#) Initial value problems for second-order hyperbolic equations
- [35L71](#) Second-order semilinear hyperbolic equations
- [35B45](#) A priori estimates in context of PDEs

Keywords:

damped wave equation; temporal decay estimates; Fourier splitting method

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