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The coupled heat Maxwell equations with temperature-dependent permittivity. (English)

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Summary: We consider the heat equation coupled with the Maxwell system, the Ampère-Maxwell equation being coupled to the heat equation by the permittivity, which depends on the temperature due to thermal agitation, and the heat equation being coupled to the Maxwell system by the volumic heat source term. Our purpose is to establish the existence of a local-in-time solution to this coupled problem. Firstly, fixing the temperature distribution, we study the resulting Maxwell system, a nonautonomous system due to the dependence of the permittivity on the temperature and consequently on time, by using the theory of evolution systems. Next, we return to our coupled problem, introducing a fixed-point problem in the closed convex set $K(0;R) := \{z \in \overline{B}(0;R); z(0) = 0\}$ of the Banach space $C^1([0,T];C^1(\overline{\Omega}))$ and proving that the hypotheses of Schauder's theorem are verified for R sufficiently large. The construction of the fixed-point problem is nontrivial as we need $K(0;R)$ to be stable.

MSC:

35Qxx Partial differential equations of mathematical physics and other areas of application
78Axx General topics in optics and electromagnetic theory
65Zxx Applications to the sciences

Keywords:

heat equation; Maxwell system; systems of evolution; coupled problem; existence theory of a local-in-time solution; Schauder's theorem

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