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**Constrained approximate null controllability of the coupled heat equation with impulse controls.** (English) [Zbl 1479.35522](#)

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The authors discuss an issue of the constrained approximate null controllability of a systems of heat equations coupled with a real matrix  $P$ . The controls acting on the system are impulsive and periodical having the form of a series of real matrices  $Q$ . It is supposed that the controls are bounded by instant constraints. Two interpretations of the considered controlled system are given. The authors give sufficient conditions of global constrained approximate null controllability in the cases where the control acts on the system globally or locally. These conditions impose certain requirements on matrices  $P$  and  $Q$ . In the case where the controls act globally, necessary conditions of such controllability are also determined.

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**MSC:**

[35K51](#) Initial-boundary value problems for second-order parabolic systems  
[93B05](#) Controllability  
[93C20](#) Control/observation systems governed by partial differential equations  
[35K90](#) Abstract parabolic equations  
[47D06](#) One-parameter semigroups and linear evolution equations  
[35R12](#) Impulsive partial differential equations

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[constrained approximate null controllability](#); [coupled heat equation](#); [impulse control](#)

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