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Approximation of radiating waves in the near-field: error estimates and application to a class of inverse problems. (English) Zbl 07403087

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Summary: Numerical approximation of radiating waves, with *a priori* truncation parameter and error estimates, is crucial for efficient simulation of forward and inverse scattering models. Convergence of a series ansatz for the wave field using the classical radiating wave functions is known only when the field is evaluated exterior to a ball circumscribing the configuration. If the configuration comprises non-convex and/or elongated scatterers, evaluation of the scattered field in the interior region of the ball is important for applications including for the far-field-data based inverse problem of identifying the scatterer boundary. In this article we develop a new error estimate for the series ansatz that facilitates identification of the truncation-parameter dependent interior convergence region. This in turn facilitates an estimate-based approach for solving the boundary identification inverse problem. We demonstrate, through numerical experiments, excellent agreement of the theoretical error estimate with respect to the truncation parameter, and the efficiency of the approach to identify scatterer shapes.

MSC:

65Nxx Numerical methods for partial differential equations, boundary value problems
78Axx General topics in optics and electromagnetic theory
35Jxx Elliptic equations and elliptic systems

Keywords:

radiating waves; far field; error estimates; inverse problem

Software:

MieSolver; TMATROM

Full Text: DOI

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