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**A fast ADI orthogonal spline collocation method with graded meshes for the two-dimensional fractional integro-differential equation.** (English) Zbl 07400546

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**Summary:** We propose and analyze a time-stepping Crank-Nicolson(CN) alternating direction implicit(ADI) scheme combined with an arbitrary-order orthogonal spline collocation (OSC) methods in space for the numerical solution of the fractional integro-differential equation with a weakly singular kernel. We prove the stability of the numerical scheme and derive error estimates. The analysis presented allows variable time steps which, as will be shown, can efficiently be selected to match singularities in the solution induced by singularities in the kernel of the memory term. Finally, some numerical tests are given.

**MSC:**

**65-XX** Numerical analysis

**35R11** Fractional partial differential equations

**45E10** Integral equations of the convolution type (Abel, Picard, Toeplitz and Wiener-Hopf type)

**65M70** Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs

**65M15** Error bounds for initial value and initial-boundary value problems involving PDEs

Cited in **3** Documents

**Keywords:**

fractional integro-differential equation; alternating direction implicit method; second-order backward differentiation formula; orthogonal spline collocation method; convergence

**Full Text:** [DOI](#)

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