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**An analytical method for determining the plastic regions around two circular holes in an infinite medium.** (English) Zbl 07398254

Appl. Math. Modelling 89, Part 1, 636-653 (2021)

Summary: The previous analytical methods mainly focused on the determination of the plastic region with only one circular hole in an infinite domain. However, there is no exact analytical method for the plastic regions when there are two circular holes. In this paper, the conformal transformation in the complex variable method is proposed to solve the elastoplastic problem of an infinite medium containing two equal circular holes. The research is based on the following preconditions: the two plastic regions formed are disconnected with each other, and each plastic region can completely surround each hole. Initially, the unknown elastic region in the physical plane is mapped onto an annular region in the image plane using the conformal transformation. Subsequently, the nonlinear equations containing the mapping function coefficients are established according to the stress continuity conditions along the two elastoplastic interfaces. The coefficients are set as the design variables and solved effectively by the differential-evolution algorithm. The presented solution is verified by comparing to the Finite Element analysis of ABAQUS software. Finally, the influences of the separation distance between the two holes and the loads on the plastic regions are analyzed.

**MSC:**

74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)
74S70 Complex-variable methods applied to problems in solid mechanics
74G10 Analytic approximation of solutions (perturbation methods, asymptotic methods, series, etc.) of equilibrium problems in solid mechanics

**Keywords:**

stress continuity condition; elastoplastic interface; differential-evolution algorithm; conformal mapping

**Software:**

ABAQUS

**Full Text:** DOI

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