Cao, Xiaodan; Oueslati, Abdelbacet; de Saxcé, Géry

**A non-incremental approach for elastoplastic plates basing on the Brezis-Ekeland-Nayroles principle.** (English) Zbl 1481.74083

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Summary: This paper is devoted to the numerical simulations of elastoplastic plates at small strains by using the non-incremental Brezis-Ekeland-Nayroles (BEN) principle. This space-time variational principle is based on the energy dissipation and transforms the boundary value problem into a null minimization one under constraints. Unlike the standard incremental procedure, the BEN principle characterizes the entire trajectory and delivers the mechanical fields at all time steps simultaneously. Two kinds of plates: the thin plate Love-Kirchhoff and the thick Reissner-Mindlin circular plates undergoing uniform pressure are studied respectively. The description of the dicretization of the principle using the mixed finite element method and the implementation are detailed. The obtained results are compared against analytical solutions provided in the literature and numerical predictions derived by the classical incremental procedure.

**MSC:**

74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)
74K20 Plates
49J52 Nonsmooth analysis

**Keywords:**

numerical simulations; plates; Brezis-Ekeland-Nayroles principle; elastoplasticity; constrained minimization

**Full Text:** DOI

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